

Lecture 05

Images & Cameras

Song Chaoyang

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A Dynamical System Approach for Softly Catching a Flying Object: Theory and Experiment

By Seyed Sina Mirrazavi Salehian; Mahdi Khoramshahi; Aude Billard (EPFL)



S. S. M. Salehian, M. Khoramshahi and A. Billard, "A Dynamical System Approach for Softly Catching a Flying Object: Theory and Experiment," in IEEE Transactions on Robotics, vol. 32, no. 2, pp. 462-471, April 2016. doi: 10.1109/TRO.2016.2536749

From Animals, to Computers, then Robots

The Concept of Vision



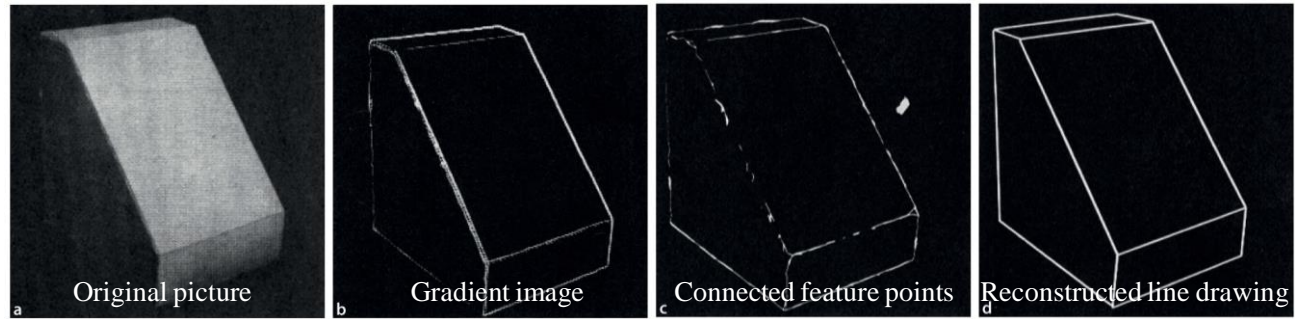
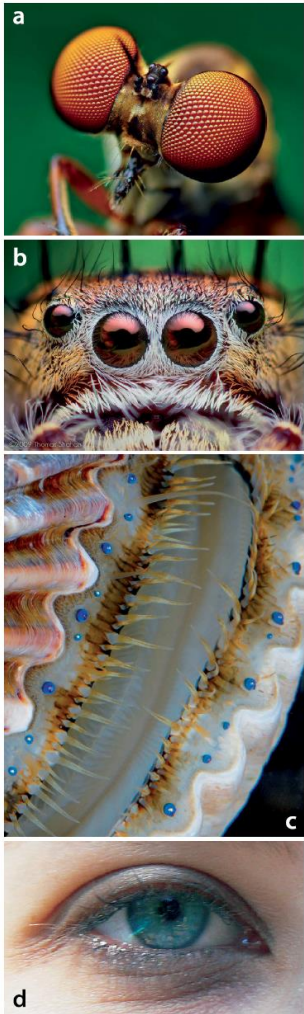
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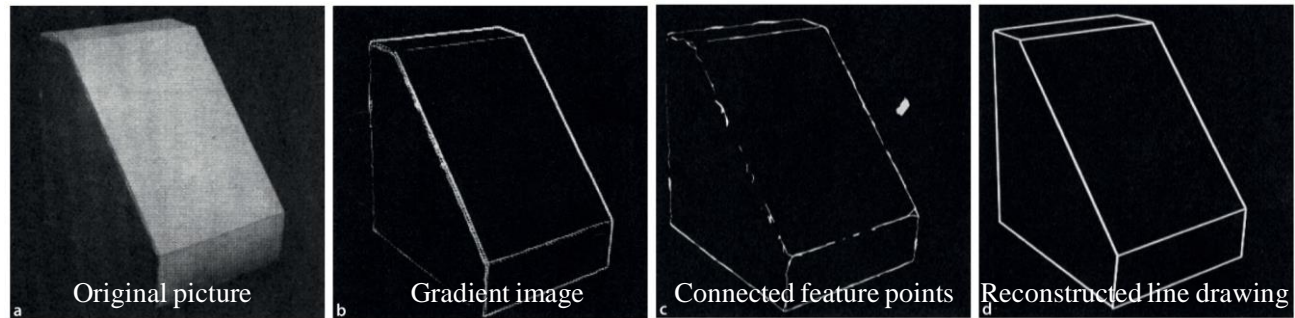
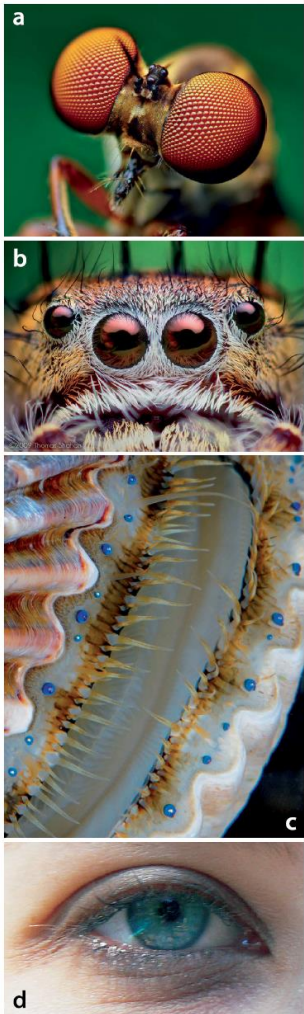
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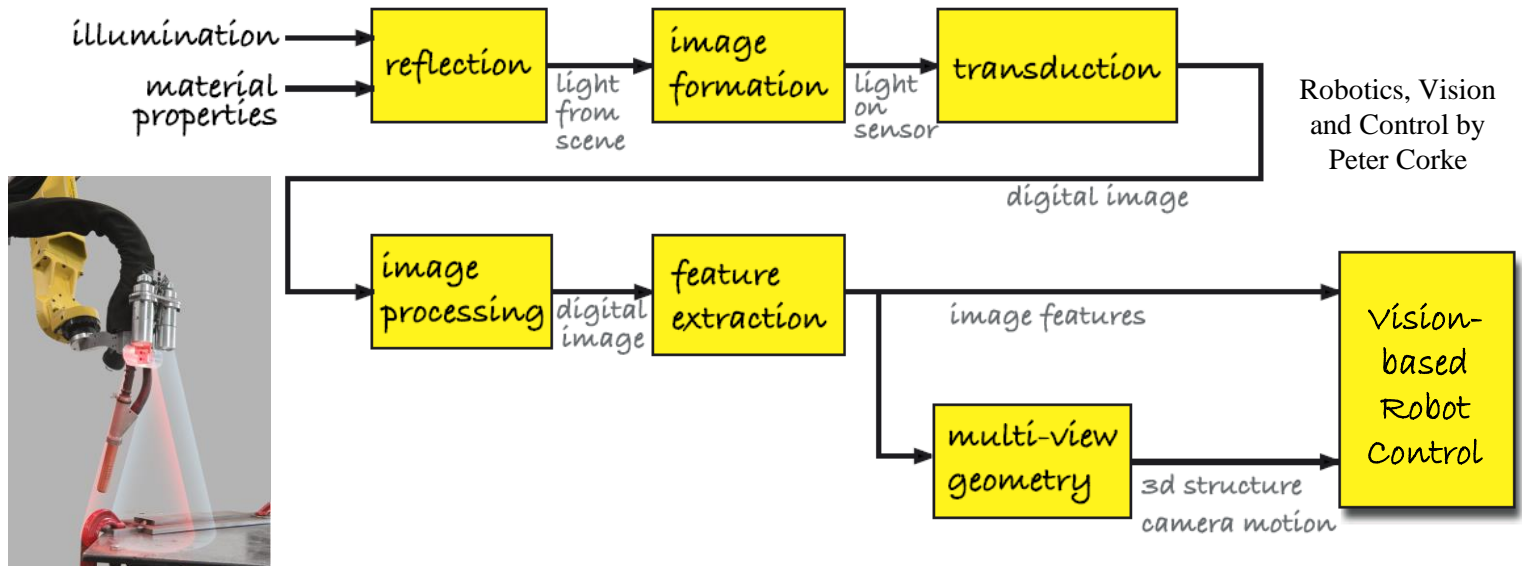
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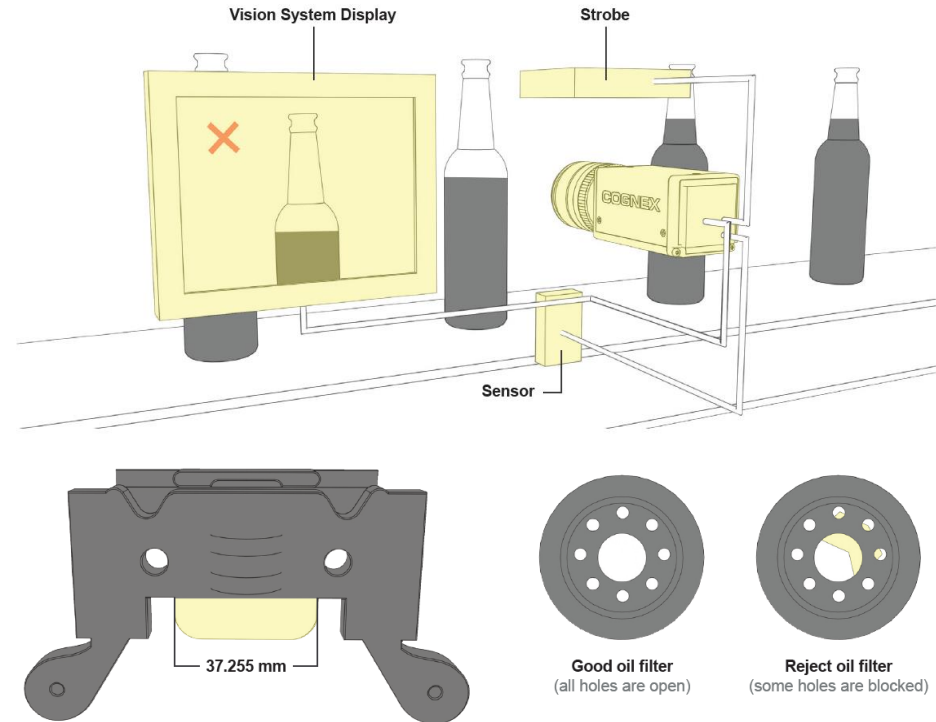
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Or the Robot as a Machine

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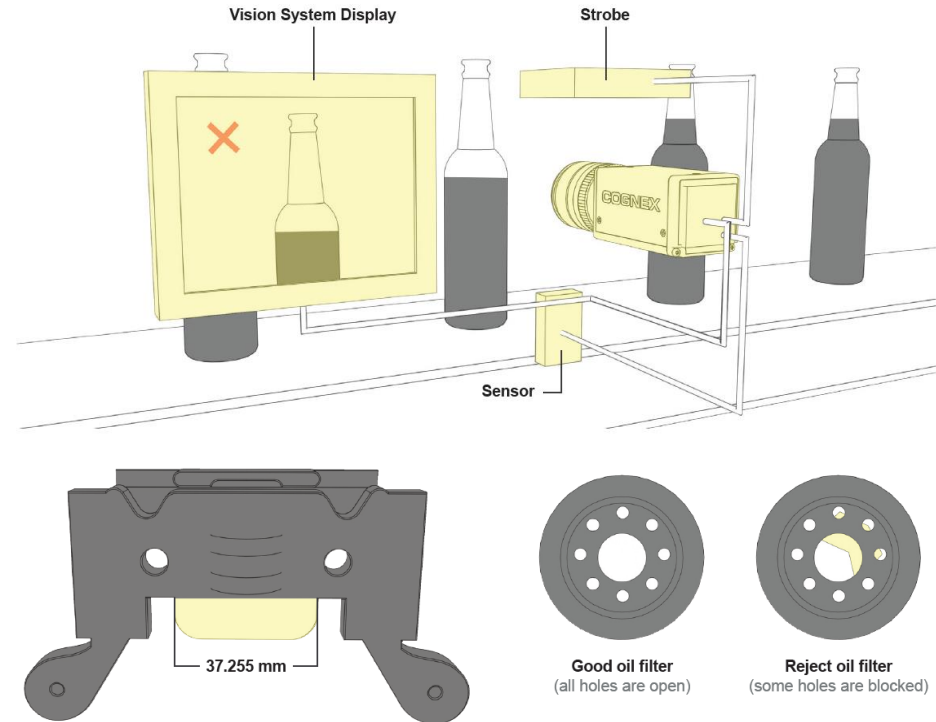
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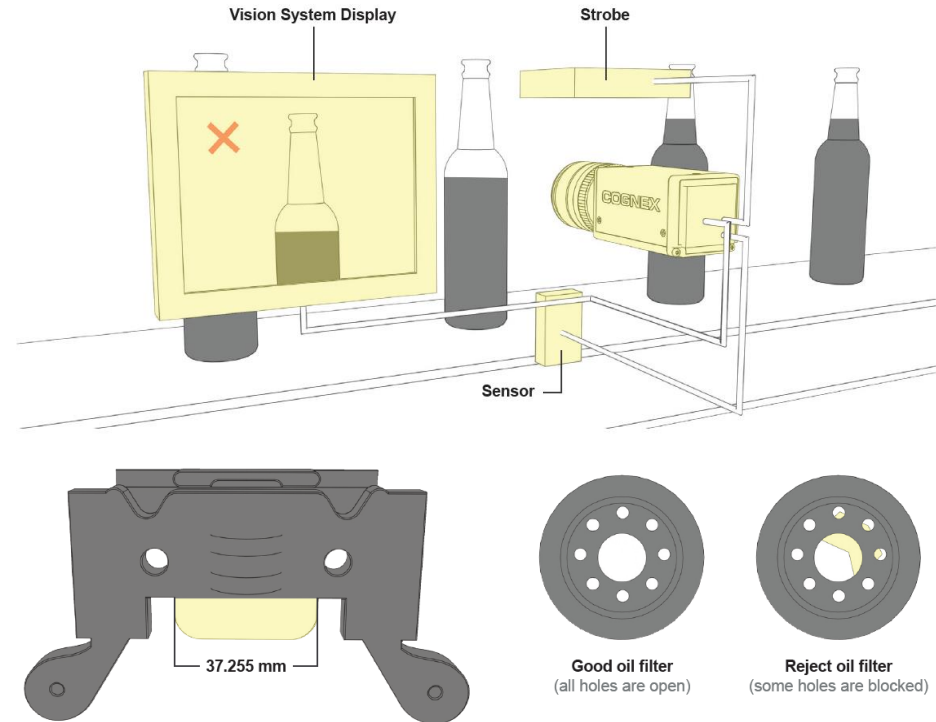
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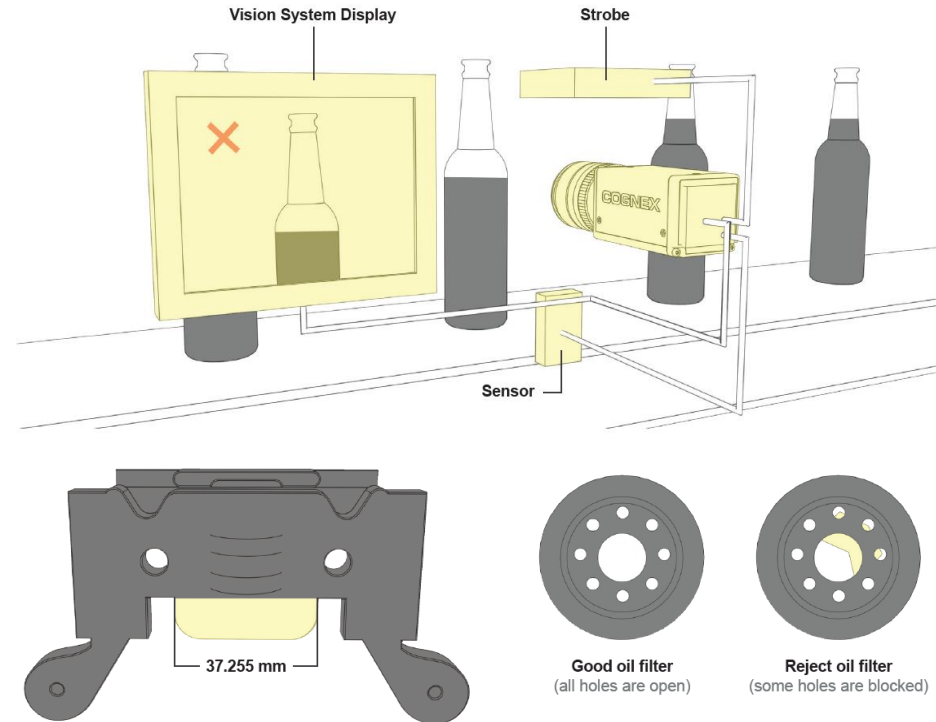
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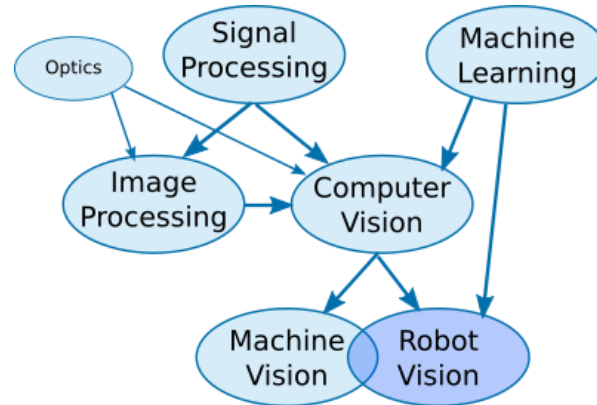
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- *An important method to interact with the physical world*

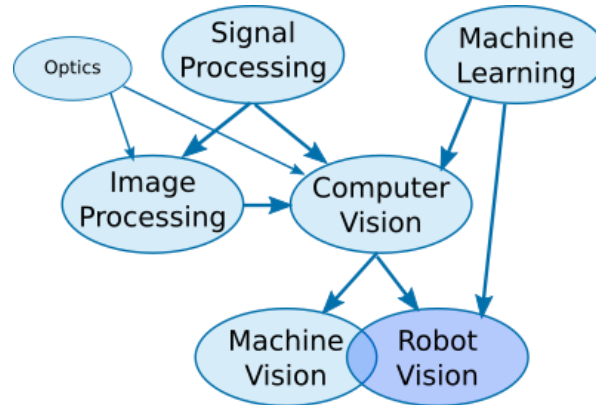


Differentiating Concepts about Vision



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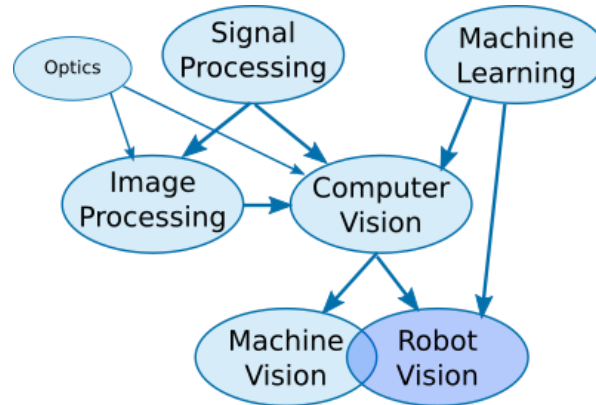
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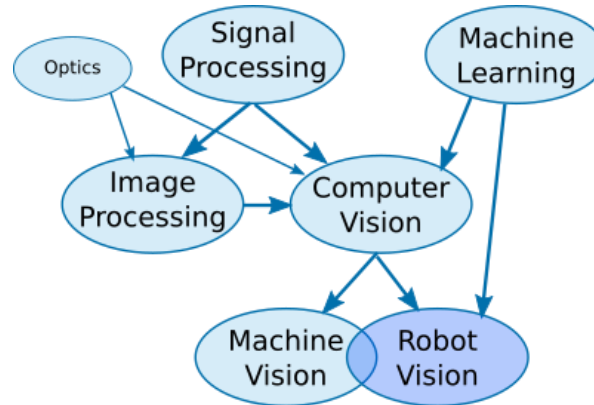


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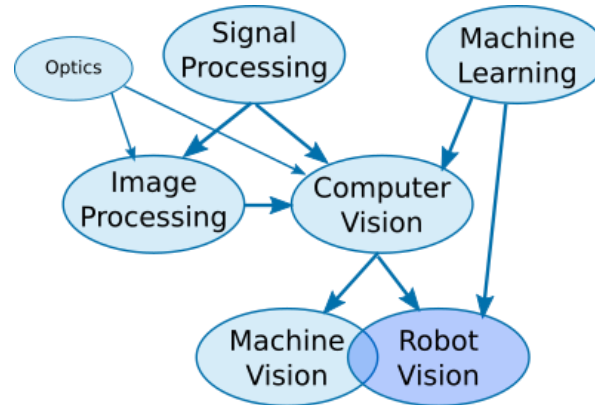


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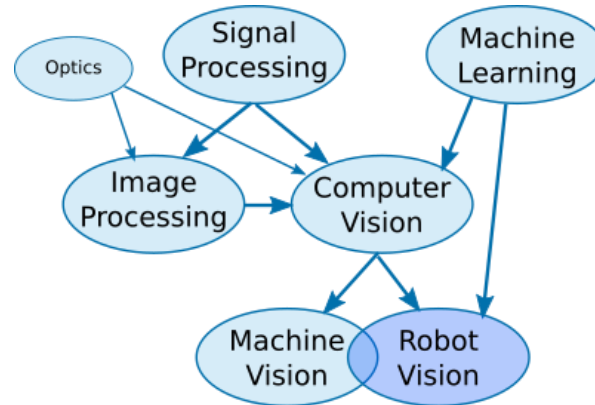
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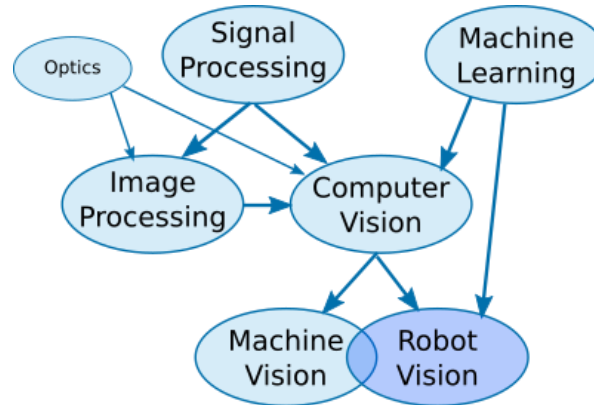
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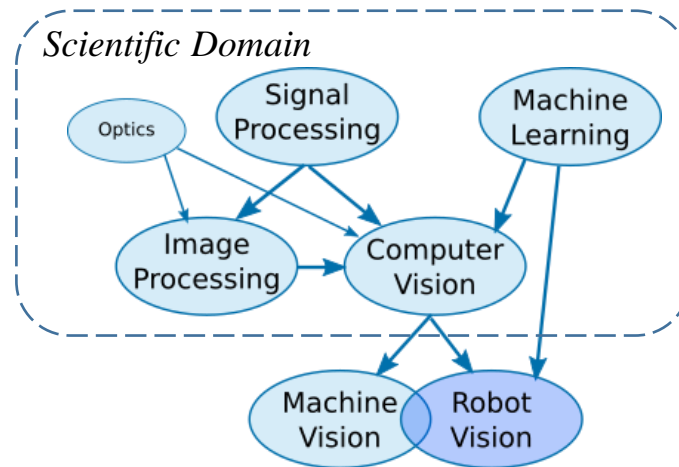
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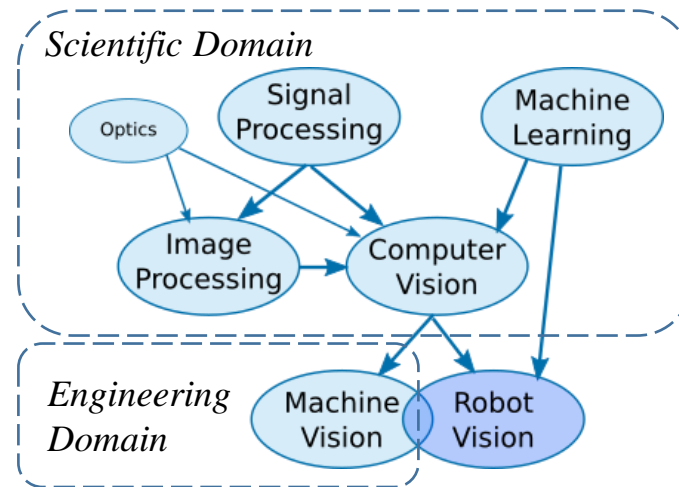
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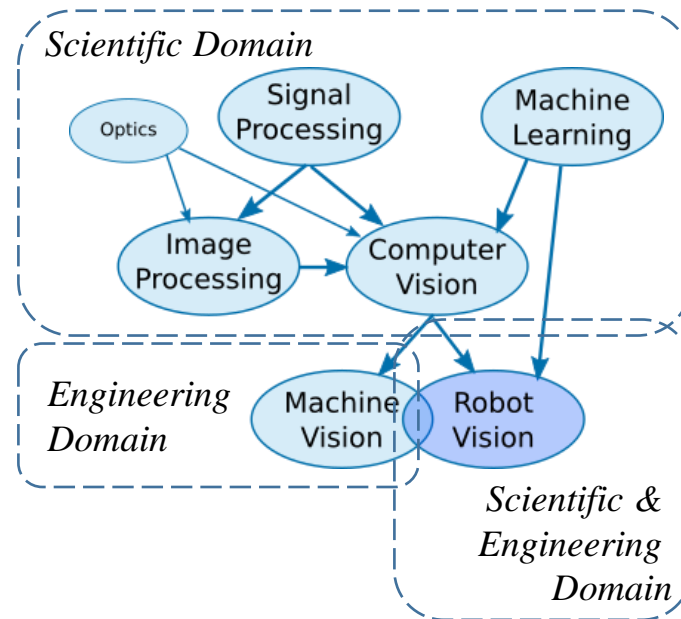
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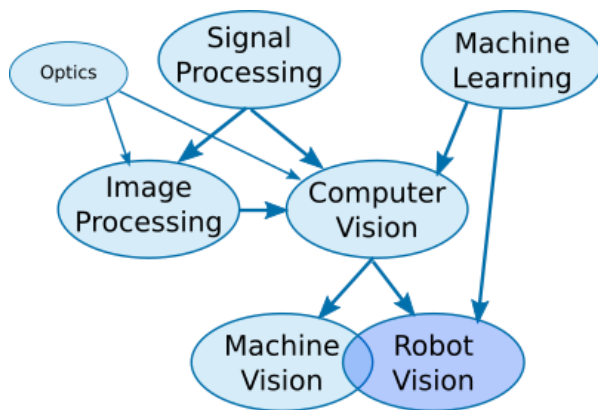
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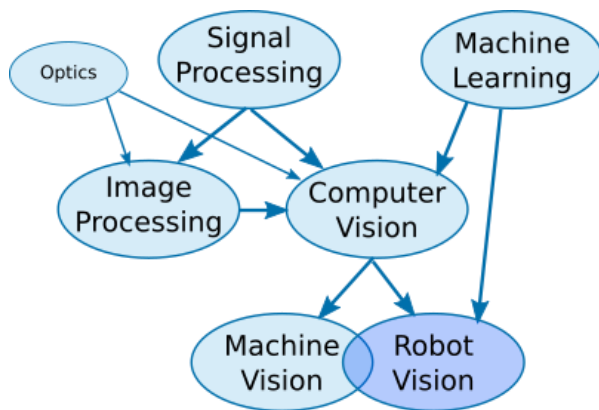
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Technique	Input	Output
Signal Processing	Electrical signals	Electrical signals
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Pattern Recognition/Machine Learning	Information/features	Information
Machine Vision	Images	Information
<i>Robot Vision</i>	Images	Physical Action



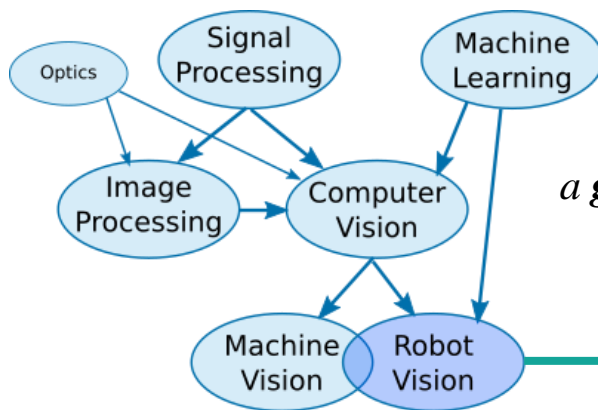
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a goal oriented machine that can sense, plan and act



Why do robots need
to see?

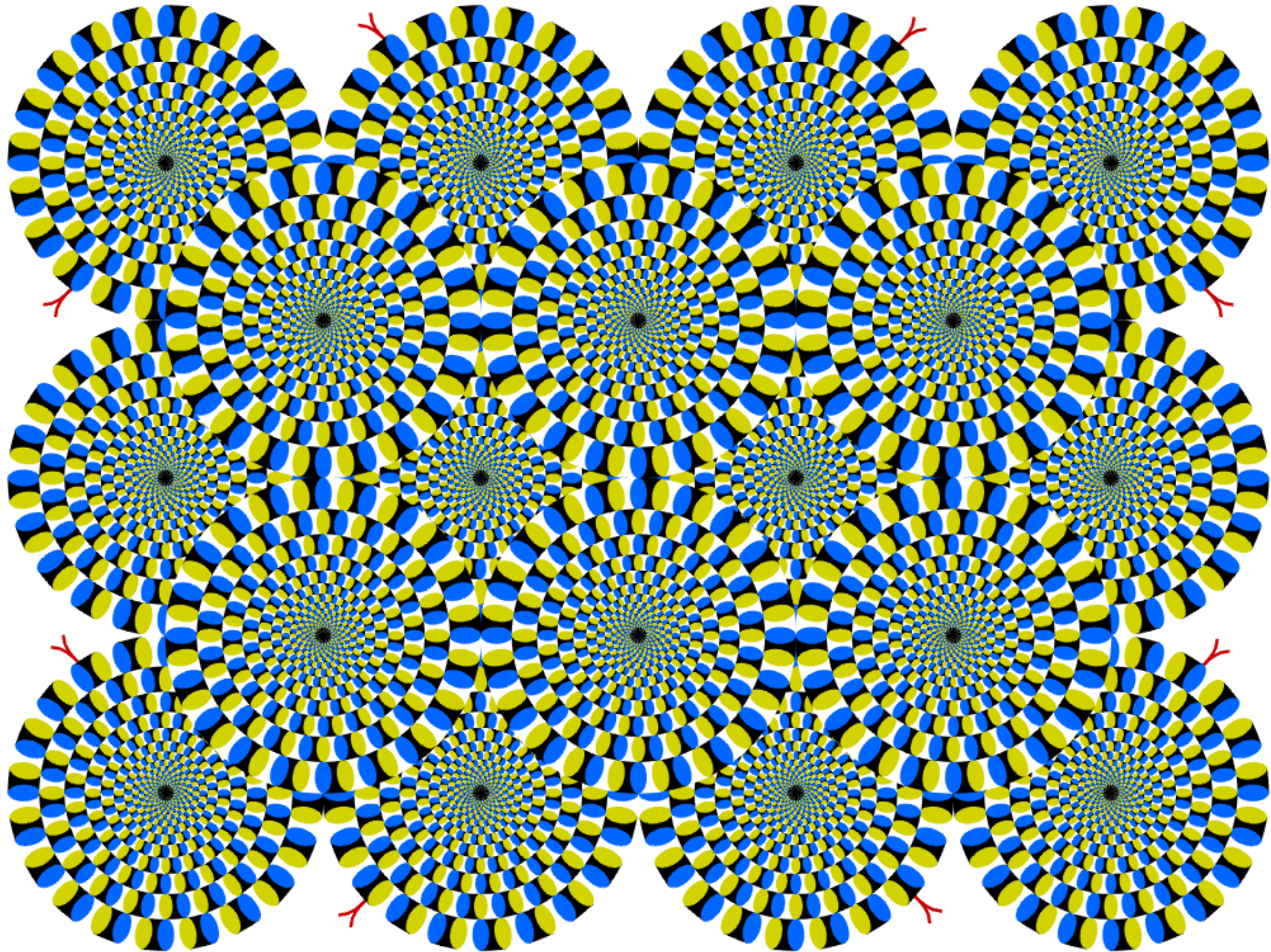


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Example of a grayscale $[0, 1]$ image within a planar area of size $[m, n]$

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p.colorbar()
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Out[3]:

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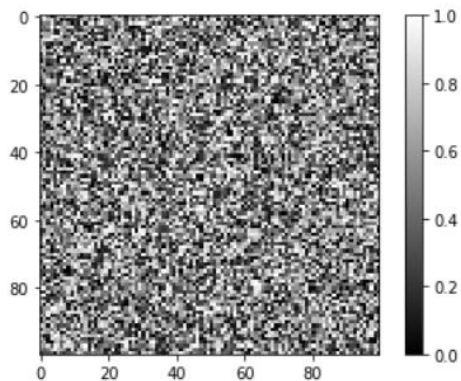


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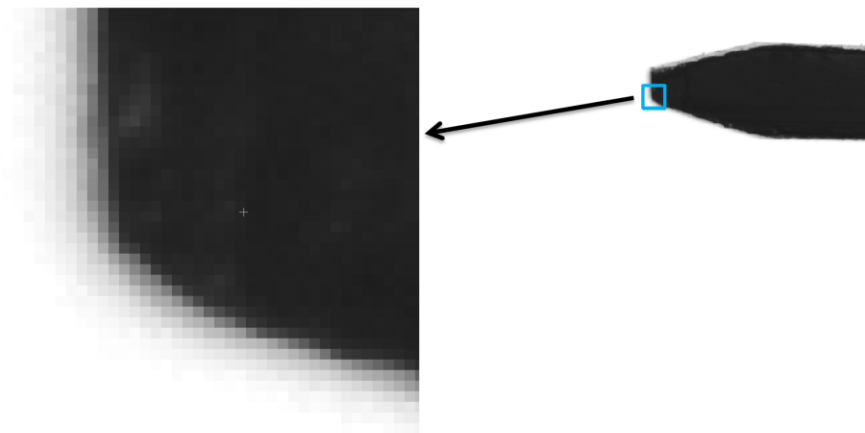
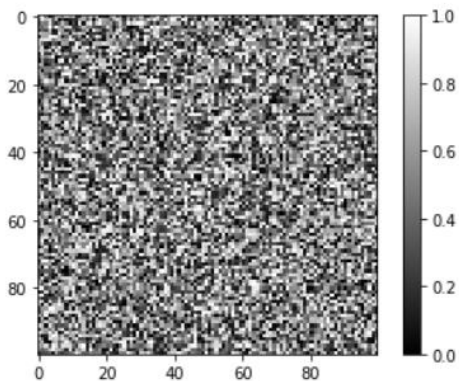


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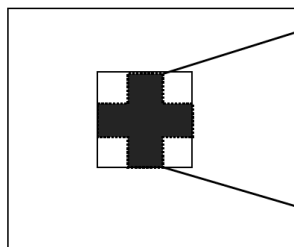
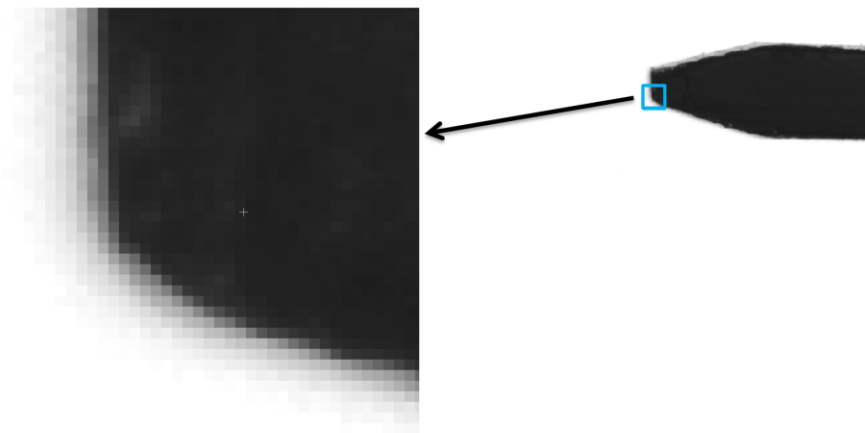
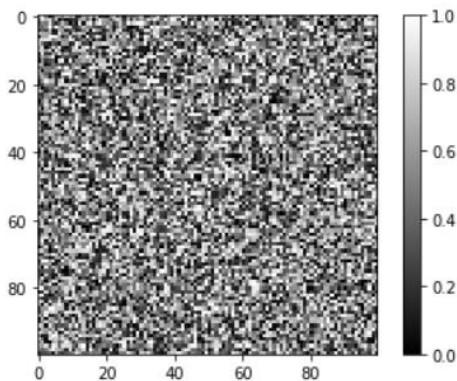
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255	255	255	116	62	44	42	57	120	255	255	255
255	255	255	112	68	41	46	58	117	255	255	255
105	110	111	109	60	42	48	61	115	112	114	108
60	68	62	57	42	41	46	41	43	49	42	41
44	42	41	46	46	42	48	44	42	42	46	42
41	46	42	48	44	42	41	41	46	43	49	42
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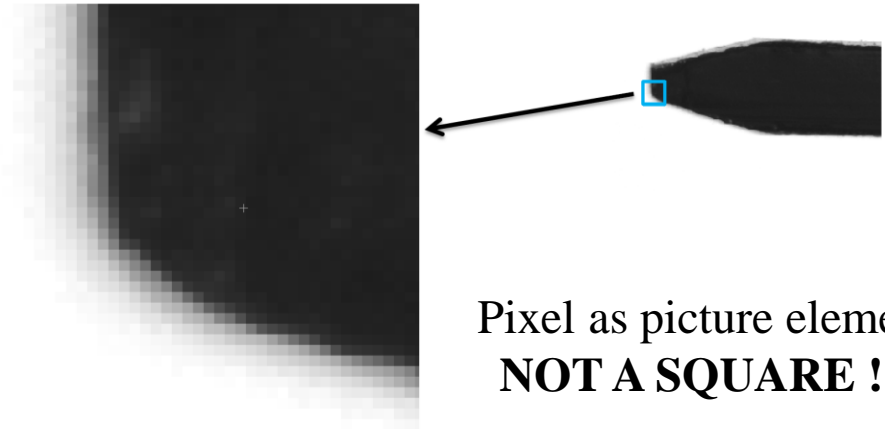
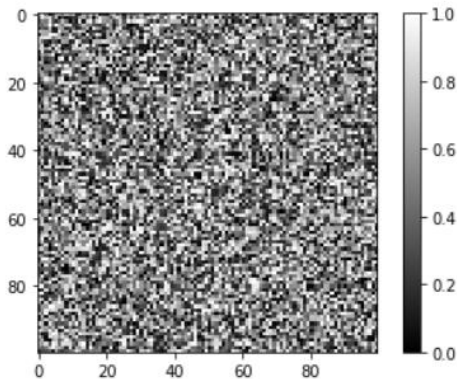
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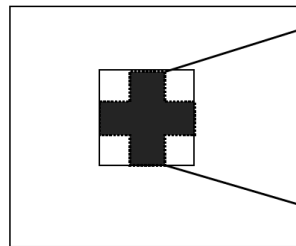
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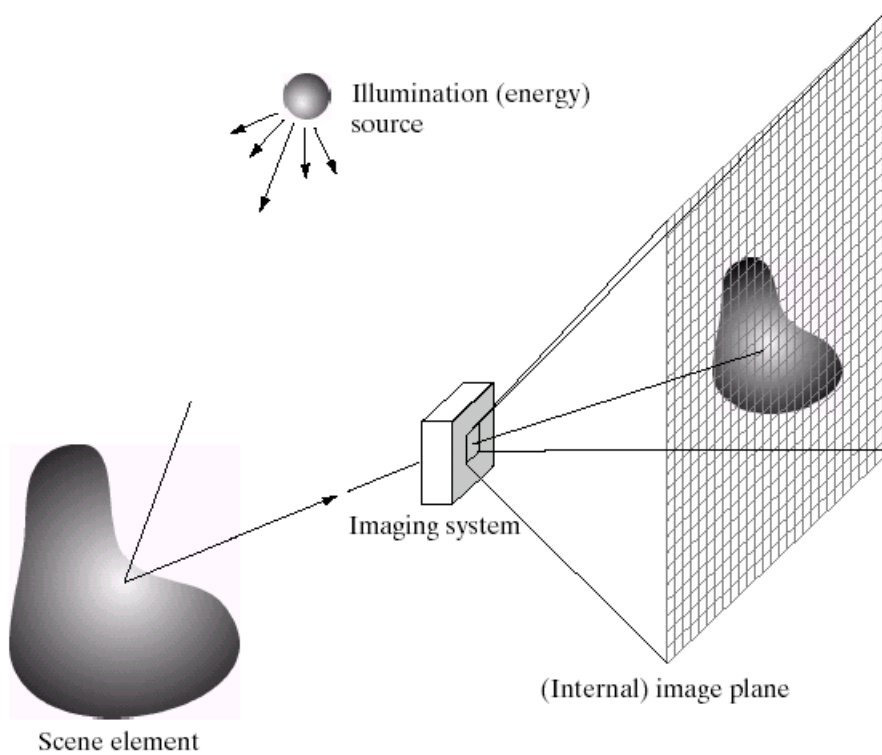
Pixel as picture element
NOT A SQUARE !!!



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Formulation of An Image

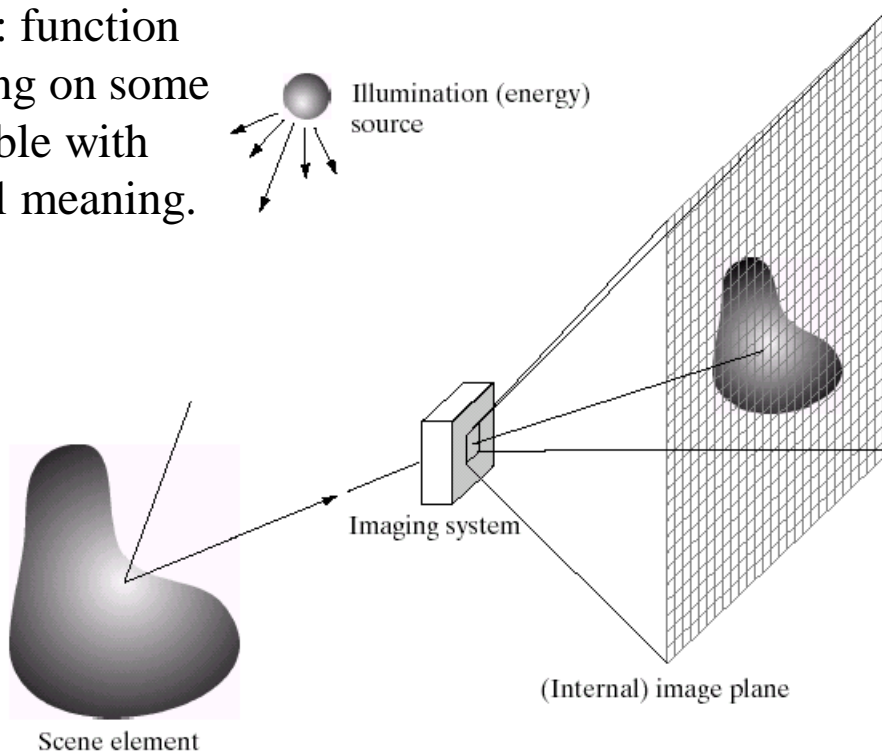
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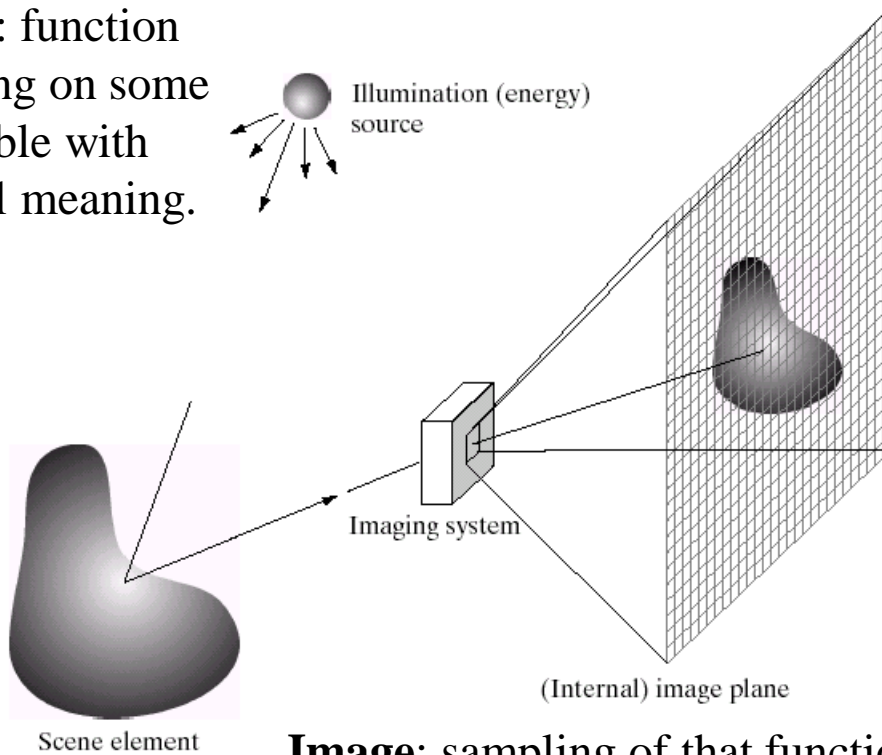


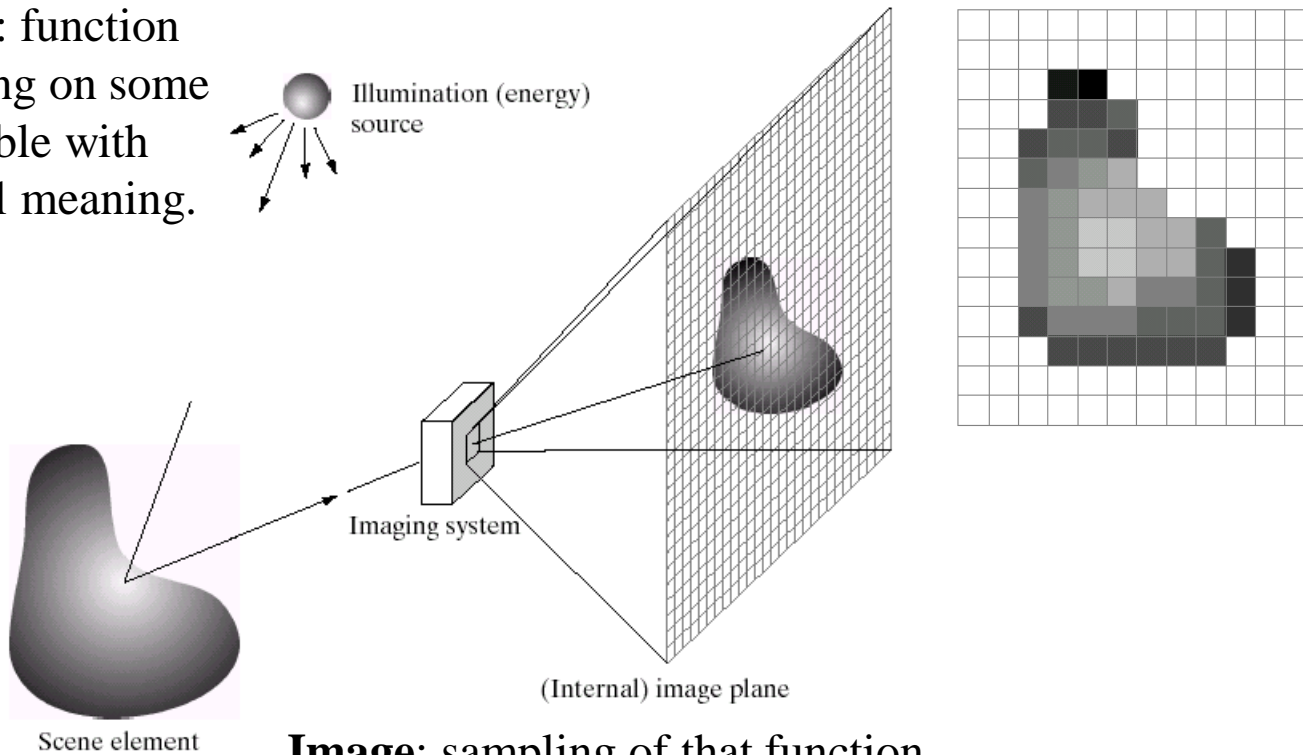
Image: sampling of that function.

- 2 variables: xy coordinates
- 3 variables: $xy + \text{time}$ (video)
- 'Brightness' is the value of the function for visible light

Formulation of An Image

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Can be other physical values too: *temperature*, *pressure*, *depth* ...

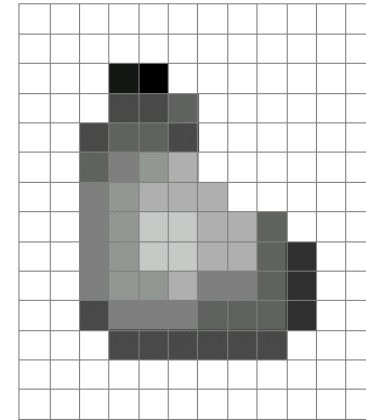
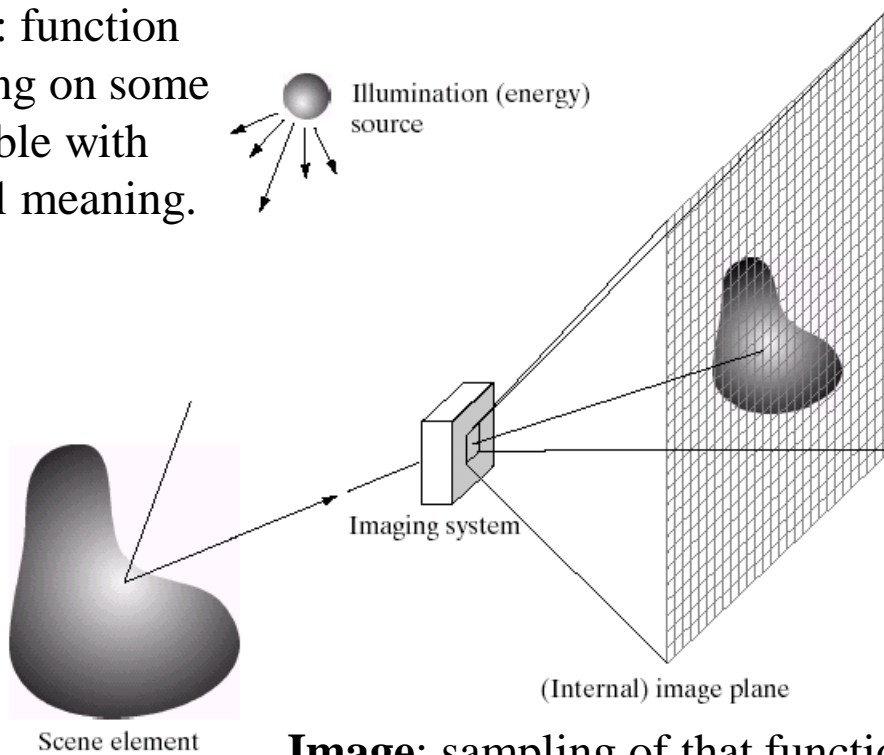
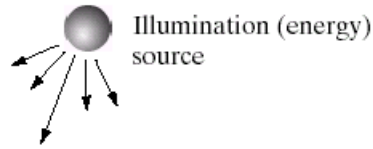
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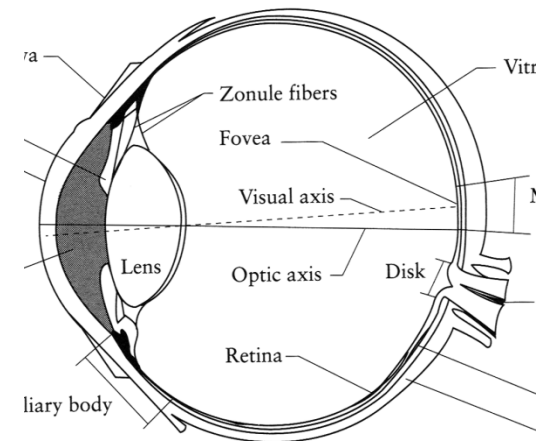
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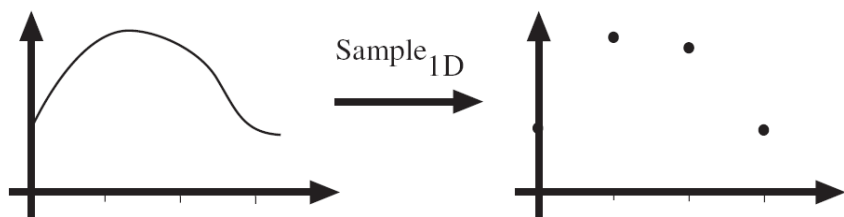
Sampling Physical World Using Images

Physical Understanding of Images



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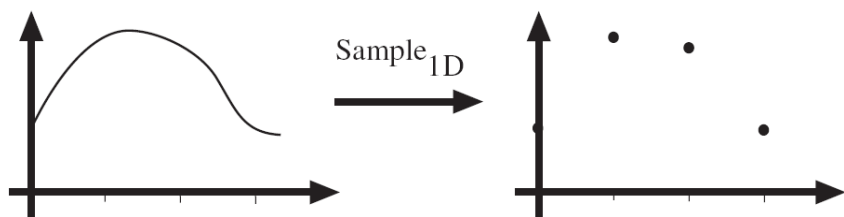
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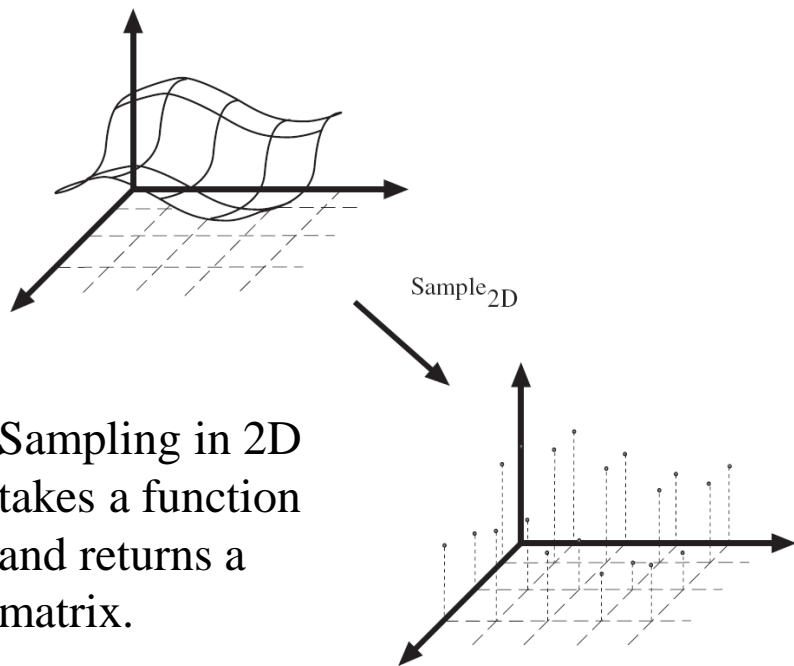
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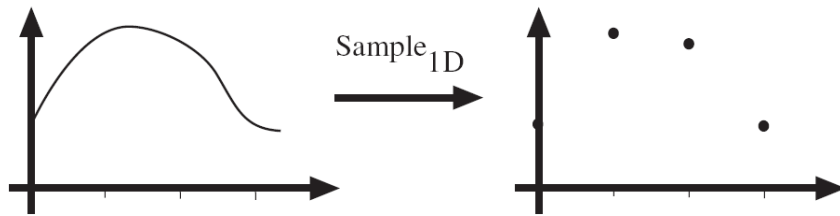


Sampling in 2D takes a function and returns a matrix.

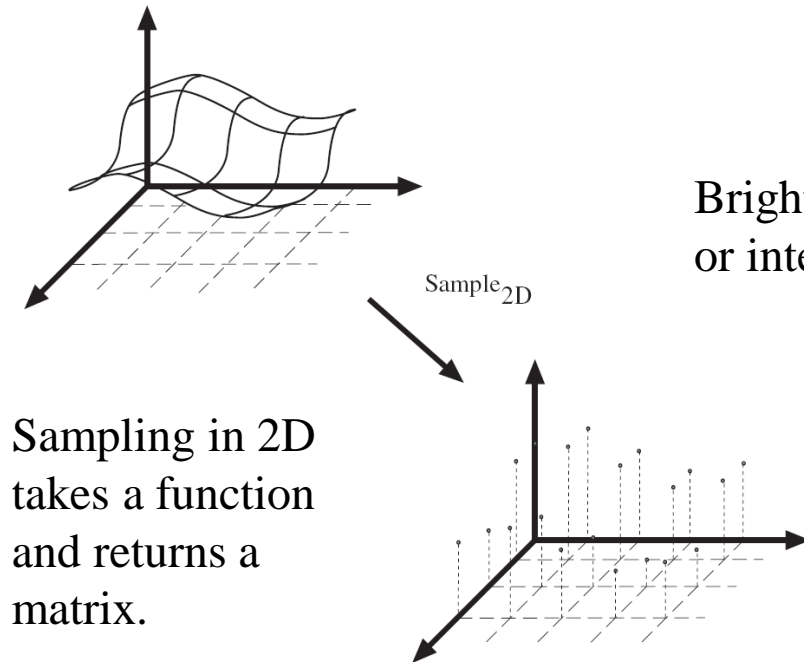


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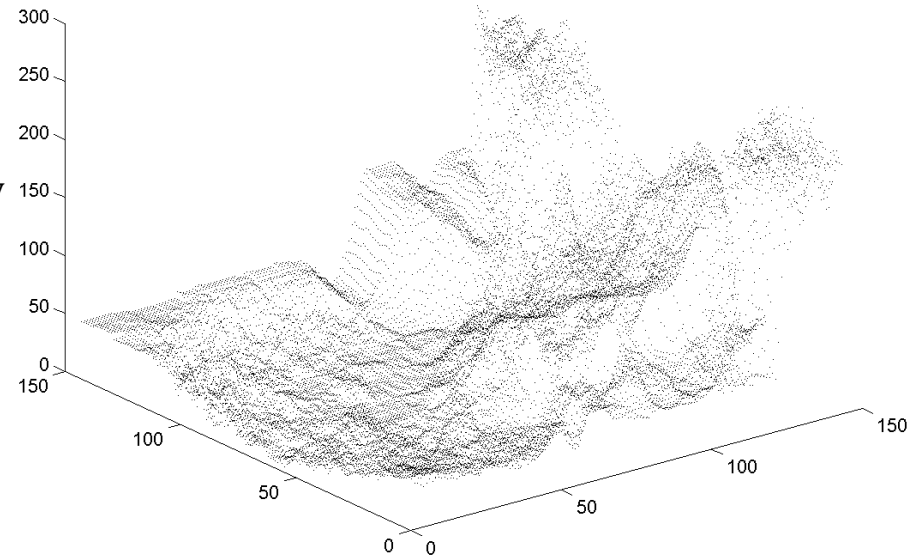
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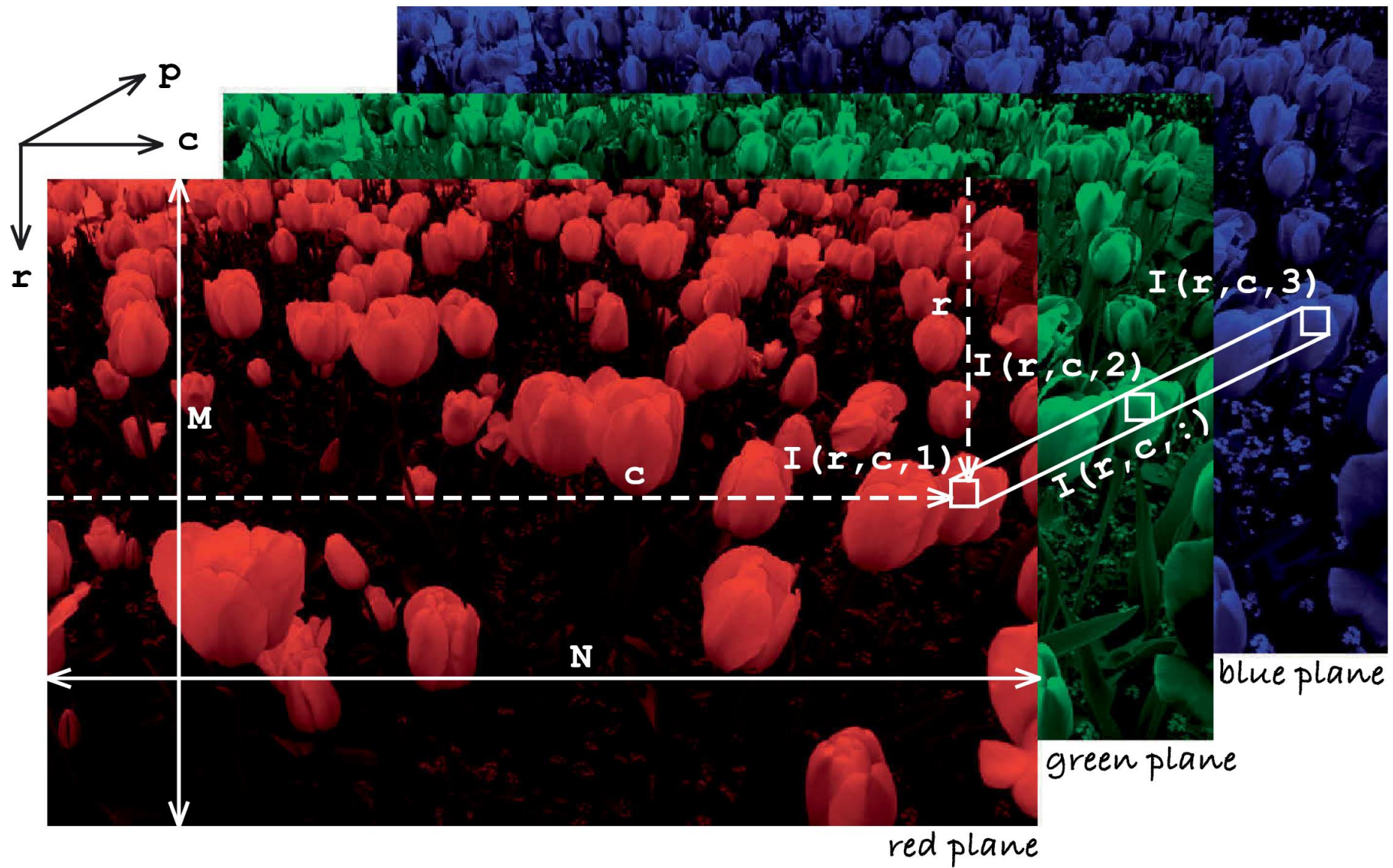
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Brightness
or intensity

Grayscale Digital Image



An RGB Image



A Robotic Way of Interpreting Images

An important method of sensing the environment



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- Computer Vision
 - Digitization of physical world in multi-dimensional linear algebra
 - *Physical meaning is not a required way of interpretation or usage*

A Robotic Way of Interpreting Images

An important method of sensing the environment

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Image Size: u, v

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Image Size: u, v

Color Space: Red, Green, Blue

Grayscale: Gray

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Image Size: u, v

Color Space: Red, Green, Blue

Grayscale: Gray

$[0, 1]$ as normalized form, not an integer
 $[0, 255]$ as a byte number of range $2^8=256$ from 0 to 255,
all in integer forms

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Image Size: u, v

Color Space: Red, Green, Blue

Grayscale: Gray

Other variables

Heatmap: H

Temperature: T

...

$[0, 1]$ as normalized form, not an integer

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Image Size: u, v

Color Space: Red, Green, Blue

Grayscale: Gray

Point Cloud: $x(u, v), y(u, v), z(u, v)$

Other variables

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Temperature: T

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Image Size: u, v

Color Space: Red, Green, Blue

Grayscale: Gray

Point Cloud: $x(u, v), y(u, v), z(u, v)$

Texture: $r(x, y, z), g(x, y, z), b(x, y, z)$

Other variables

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Image Size: u, v

Time Series: t

Other variables

Color Space: Red, Green, Blue

Grayscale: Gray

Heatmap: H

Temperature: T

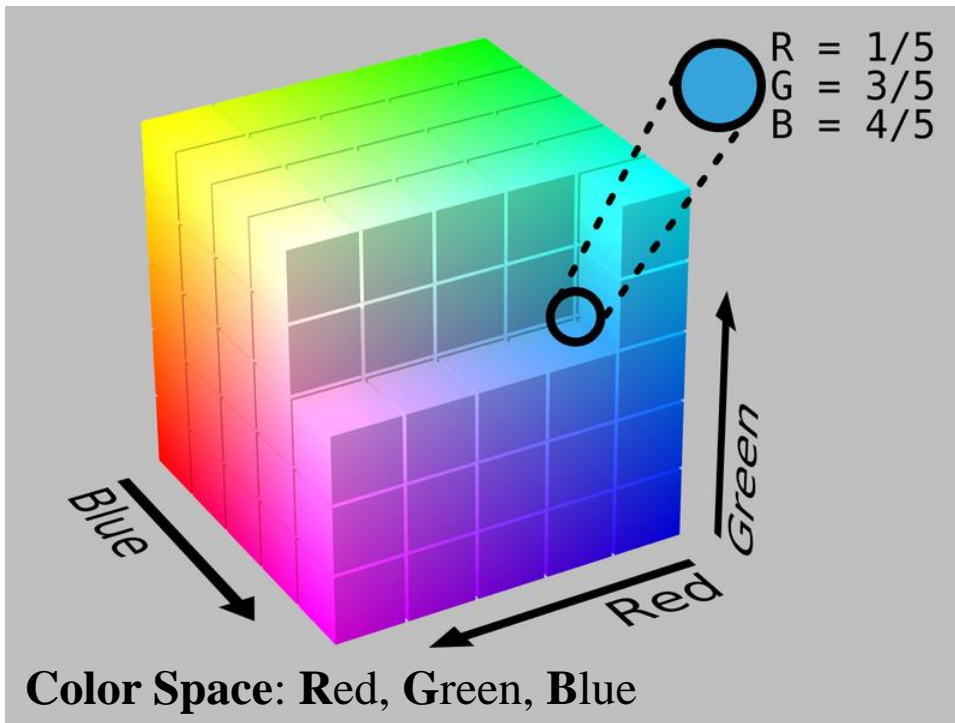
...

Point Cloud: $x(u, v), y(u, v), z(u, v)$

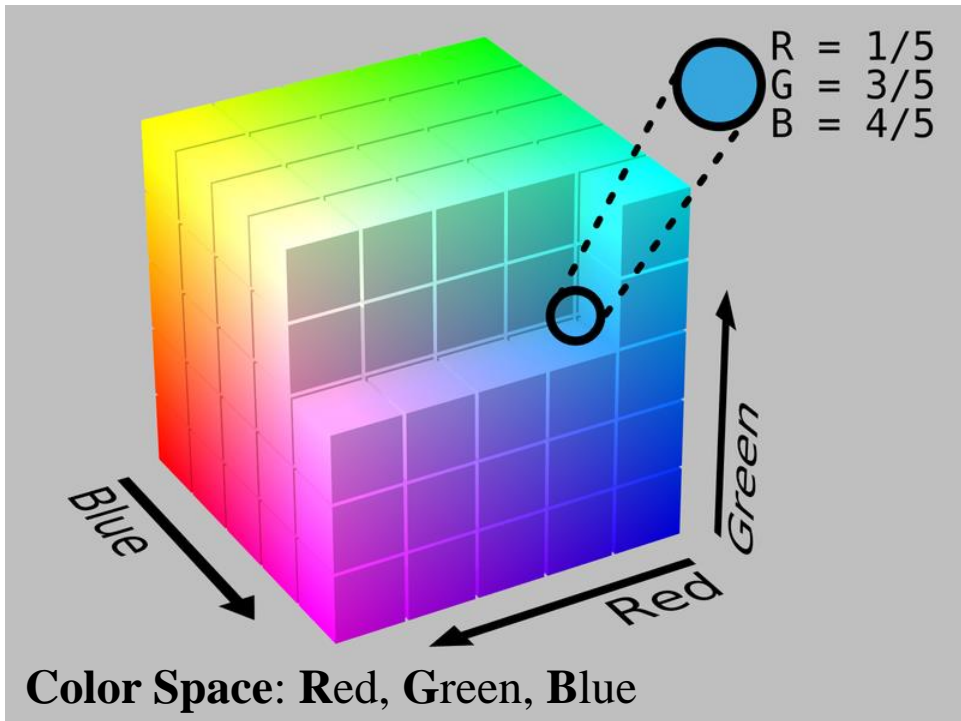
Texture: $r(x, y, z), g(x, y, z), b(x, y, z)$

$[0, 1]$ as normalized form, not an integer

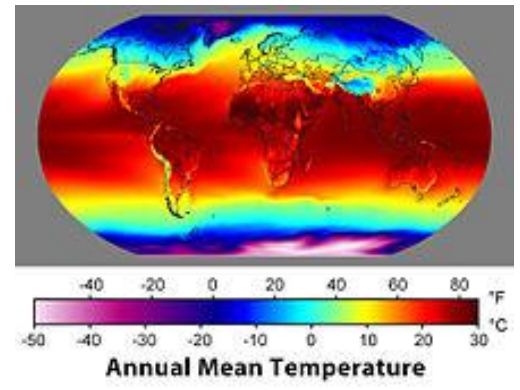
$[0, 255]$ as a byte number of range $2^8=256$ from 0 to 255, all in integer forms

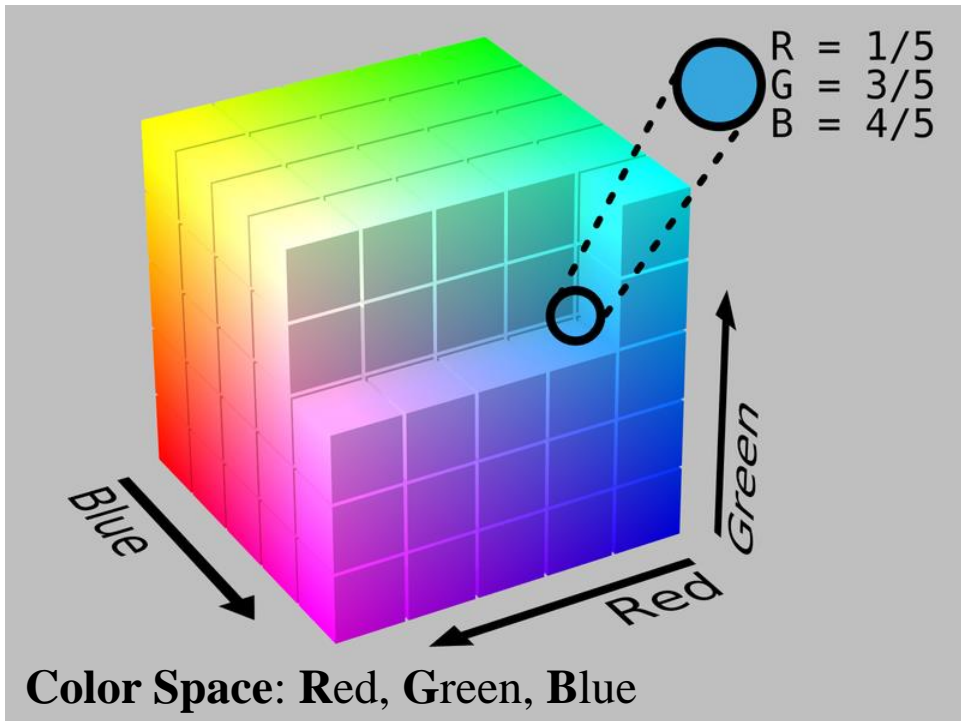


Color Space: Red, Green, Blue

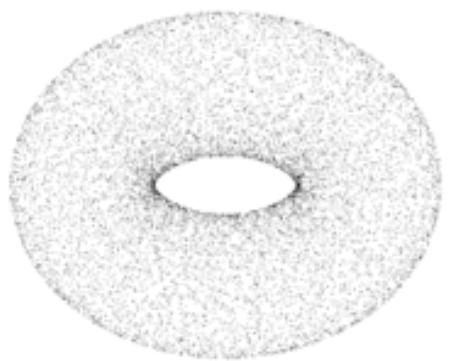
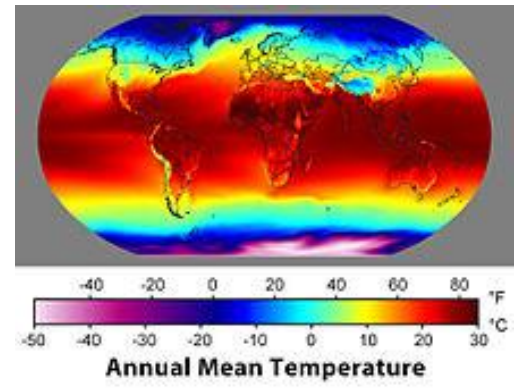


Other variables
Heatmap: H
Temperature: T
...

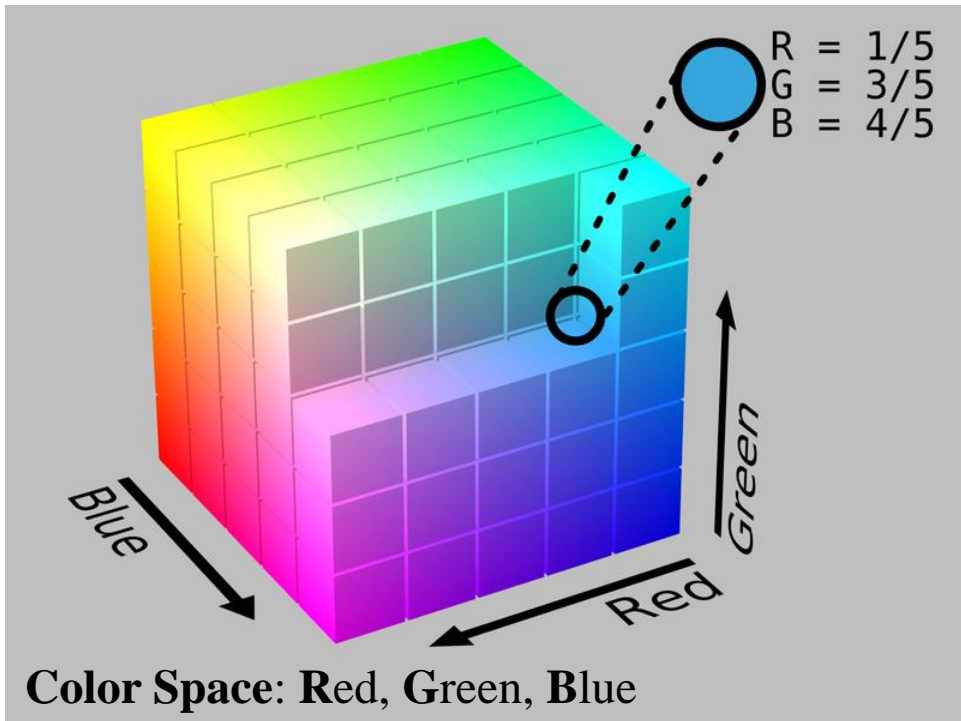




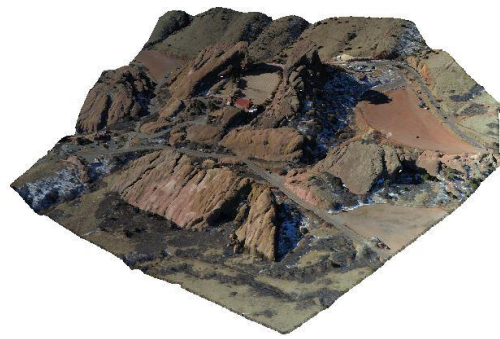
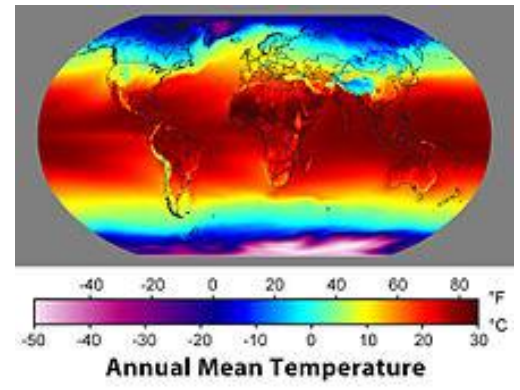
Other variables
Heatmap: H
Temperature: T
...



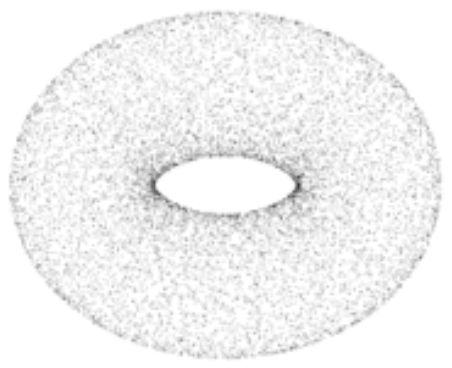
Point Cloud: $x(u, v), y(u, v), z(u, v)$
AncoraSIR.com



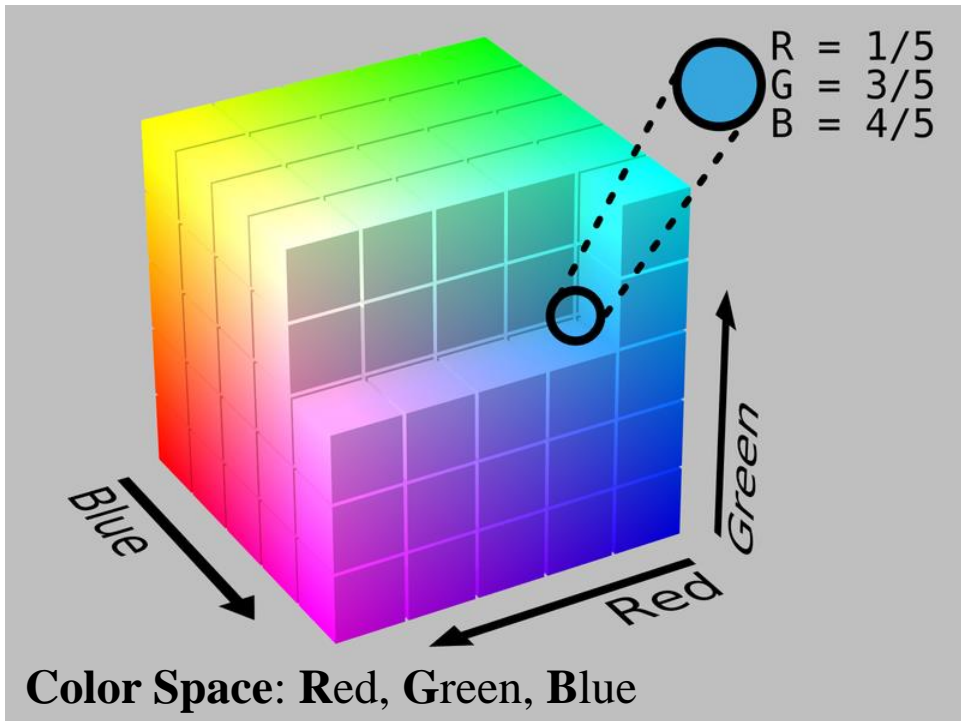
Other variables
Heatmap: H
Temperature: T
...



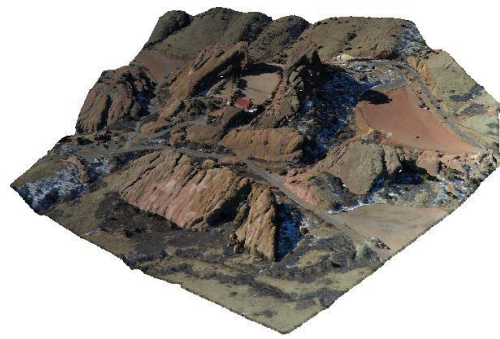
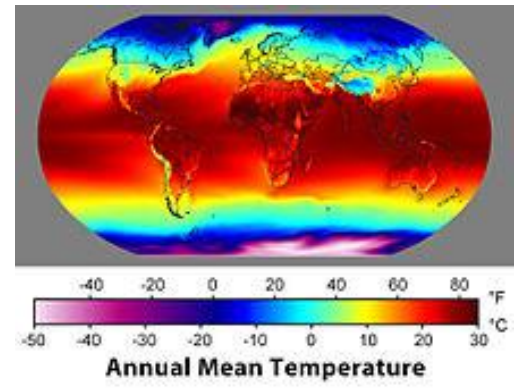
Texture:
 $r(x, y, z)$,
 $g(x, y, z)$,
 $b(x, y, z)$



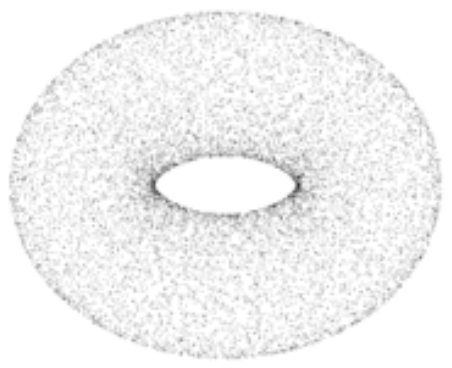
Point Cloud: $x(u, v), y(u, v), z(u, v)$
AncoraSIR.com



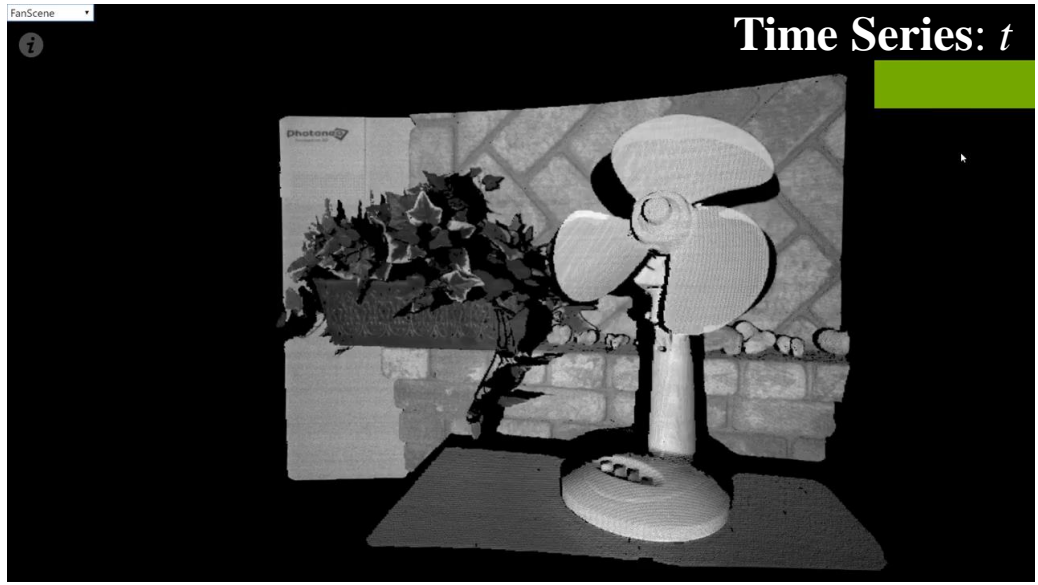
Other variables
Heatmap: H
Temperature: T
...



Texture:
 $r(x, y, z)$,
 $g(x, y, z)$,
 $b(x, y, z)$

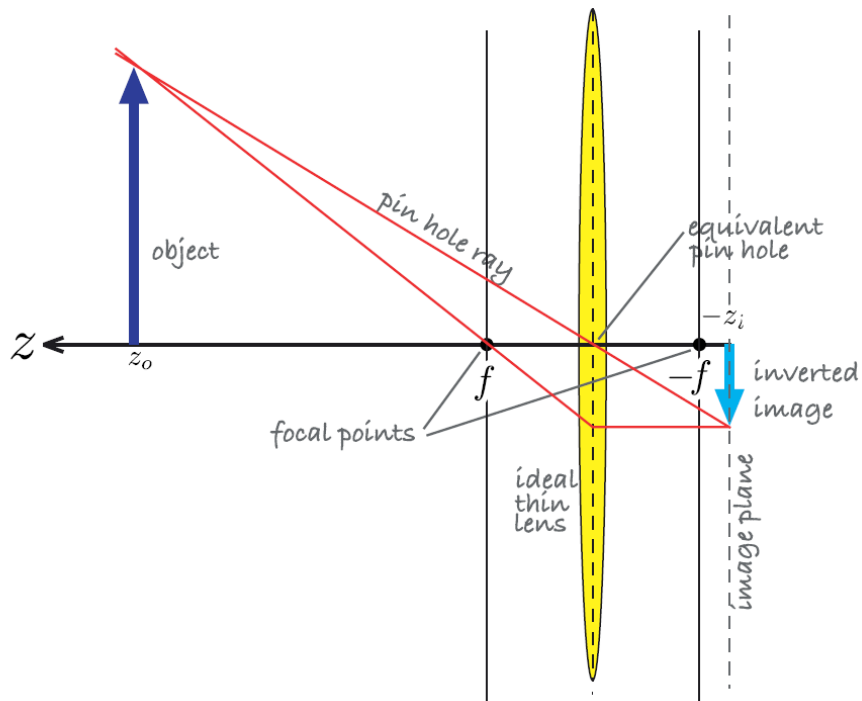


Point Cloud: $x(u, v)$, $y(u, v)$, $z(u, v)$
AncoraSIR.com



Perspective Transform

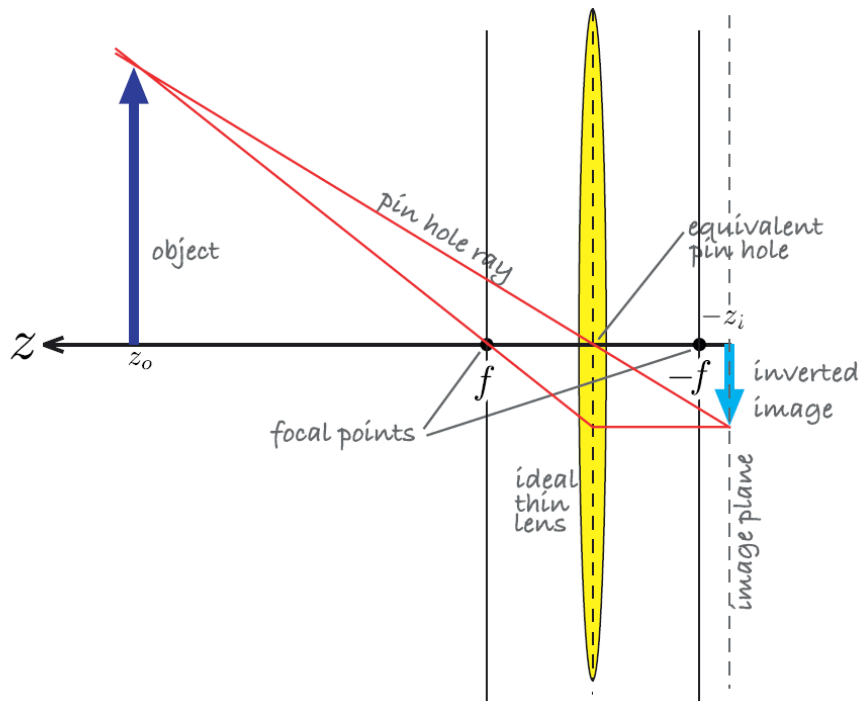
Camera Models



Perspective Transform

Camera Models

Lens Law $\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$

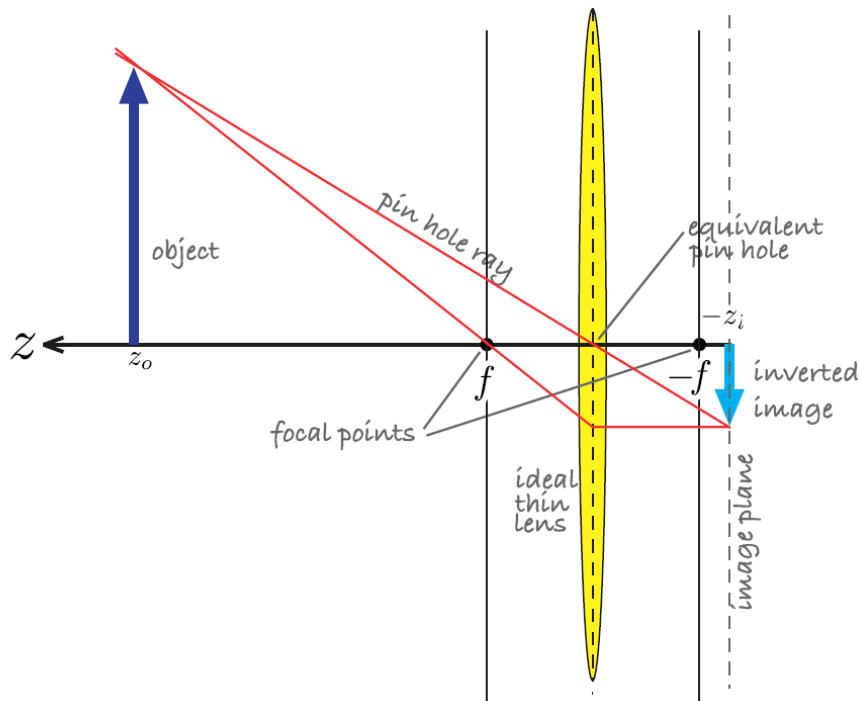


Perspective Transform

Camera Models

Lens Law $\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$

the distance
to the object



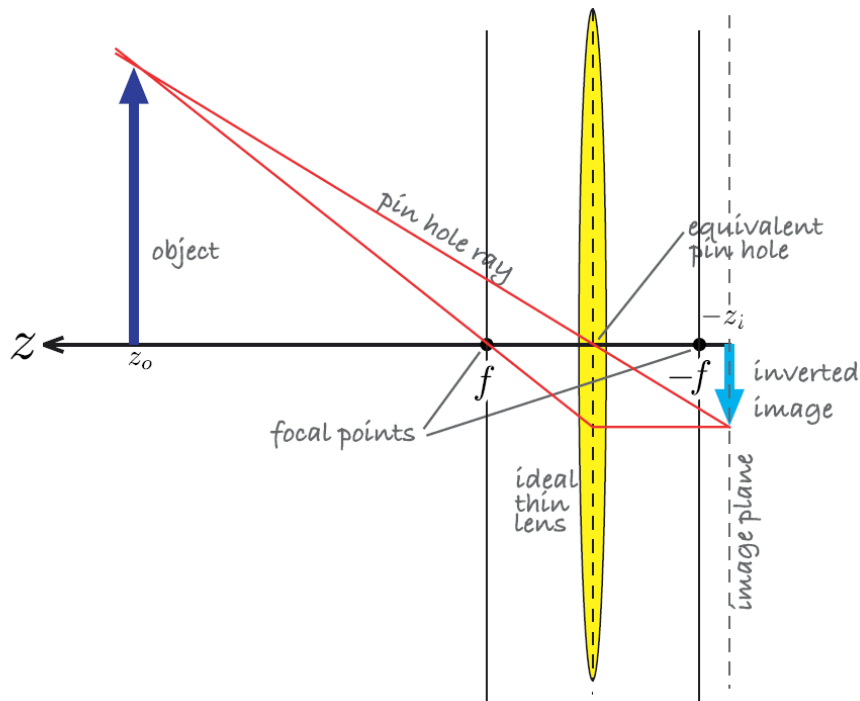
Perspective Transform

Camera Models

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the distance
to the object

the distance
to the image



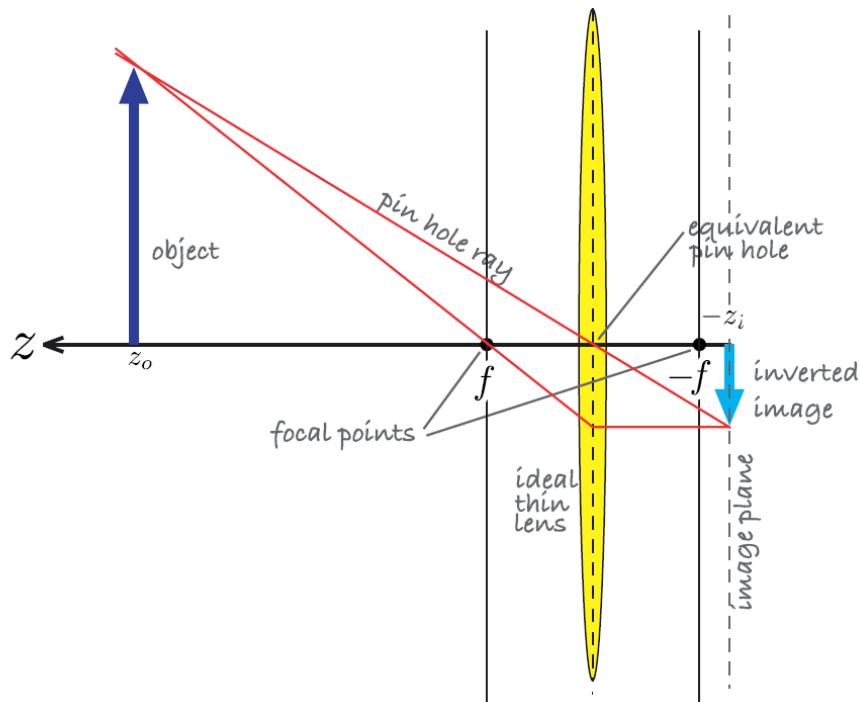
Perspective Transform

Camera Models

Lens Law $\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$

the distance to the object z_o the distance to the image z_i

the focal length of the lens f



Perspective Transform

Camera Models

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the distance to the object z_o the distance to the image z_i

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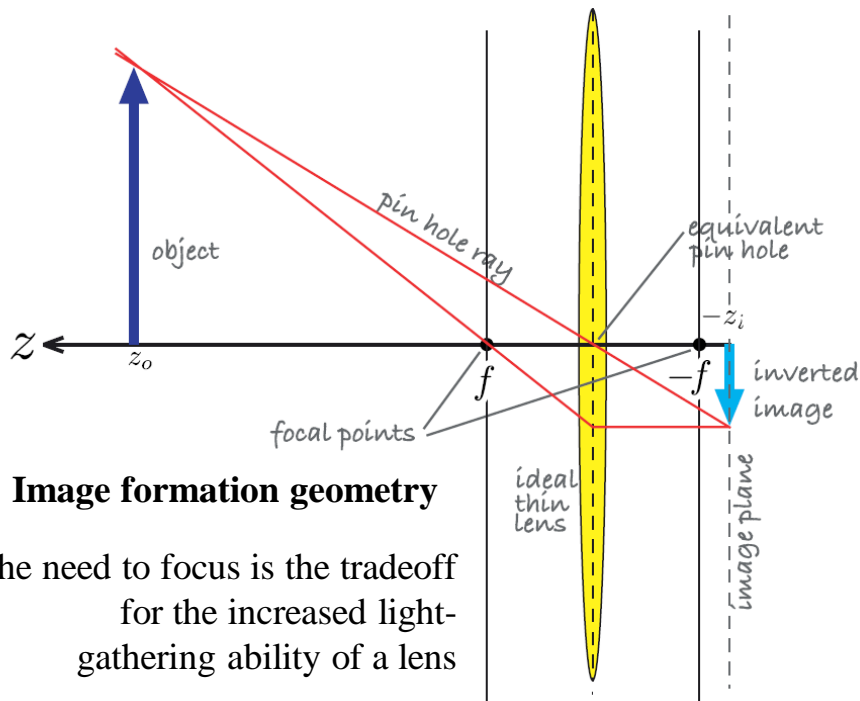


Image formation geometry

the need to focus is the tradeoff for the increased light-gathering ability of a lens

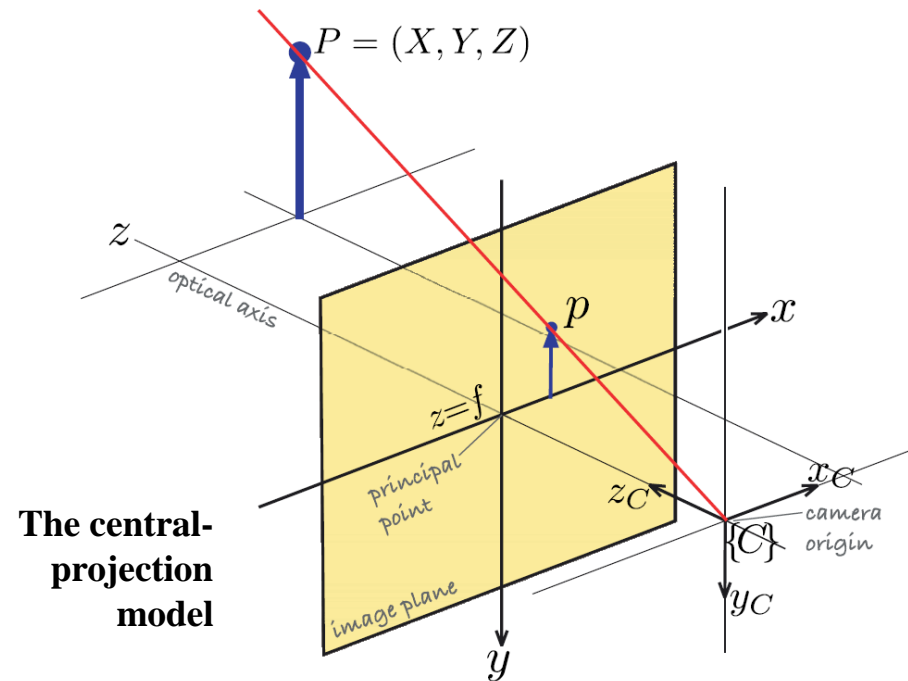
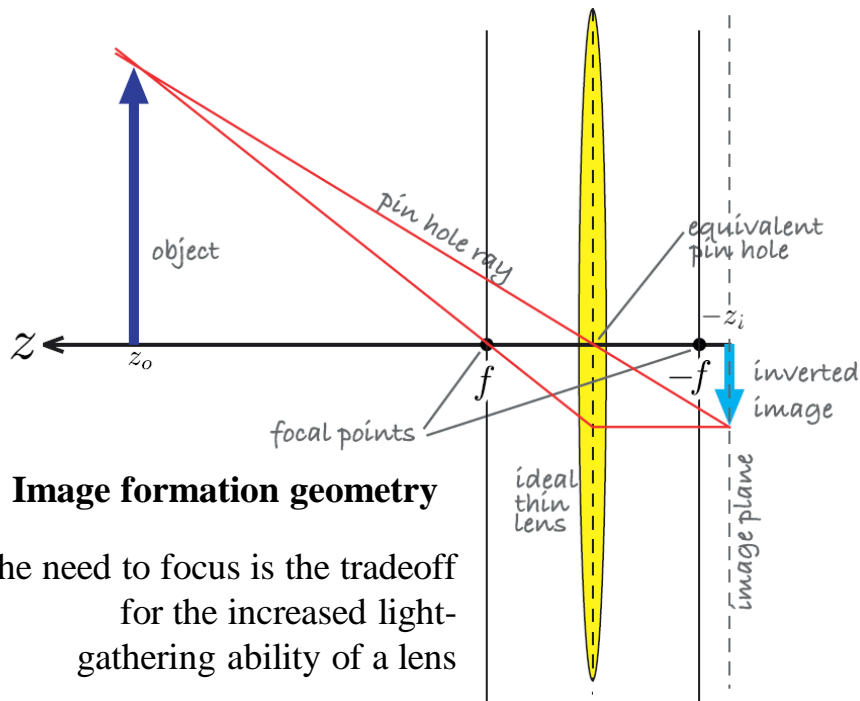
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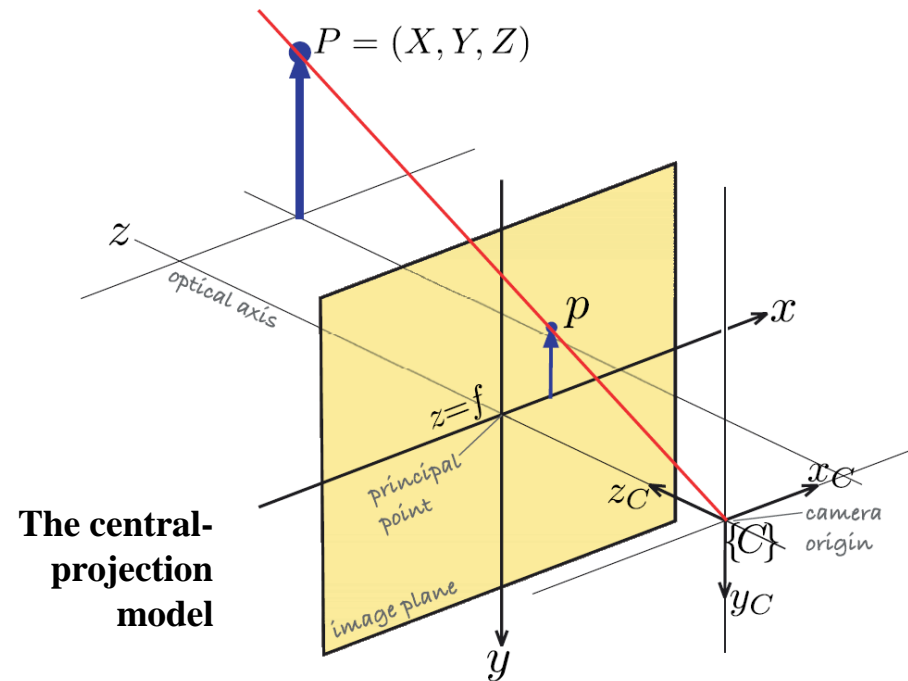
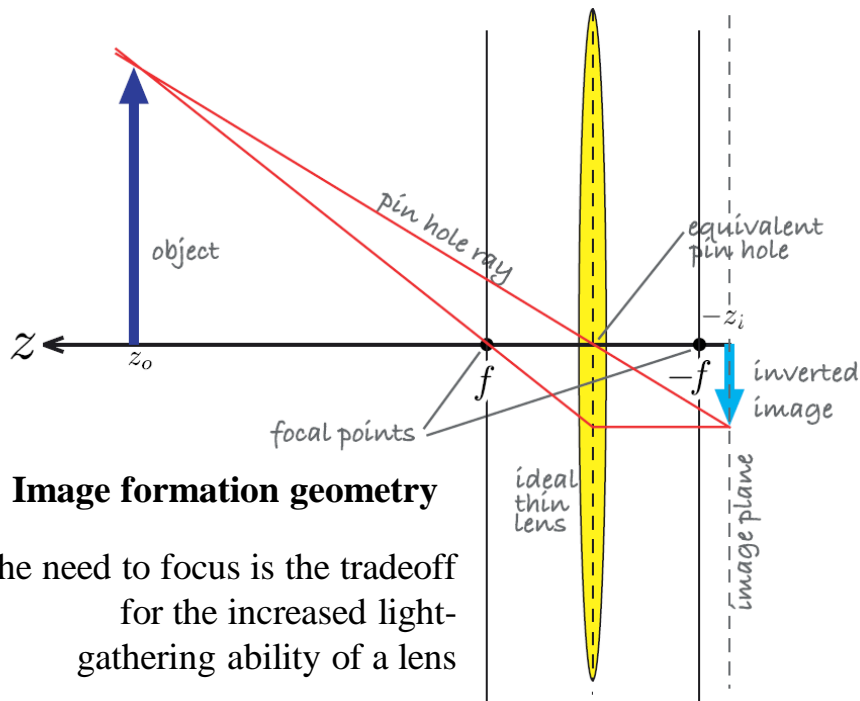
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the distance to the object z_o the distance to the image z_i

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perspective projection $x = f \frac{X}{Z}, y = f \frac{Y}{Z}$



Perspective Transform

Camera Models

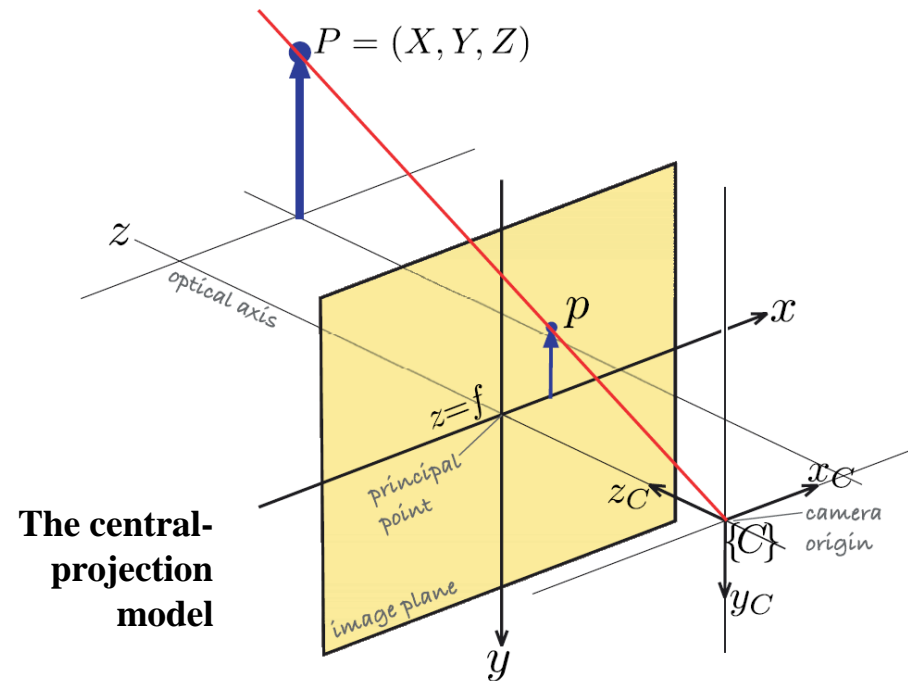
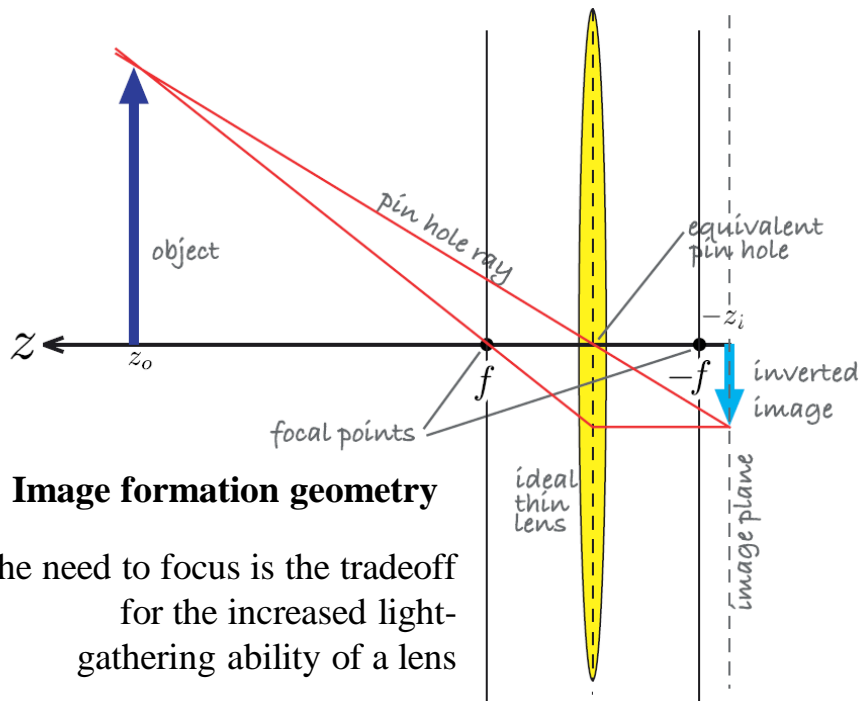
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the focal length of the lens f

perspective projection $x = f \frac{X}{Z}, y = f \frac{Y}{Z}$

$$P = (X, Y, Z)$$



Perspective Transform

Camera Models

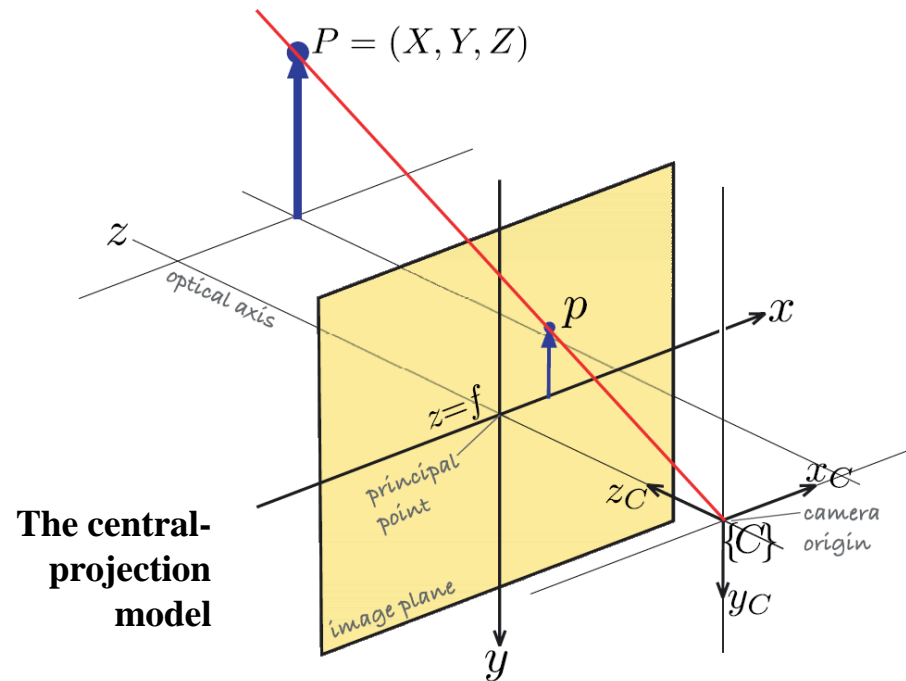
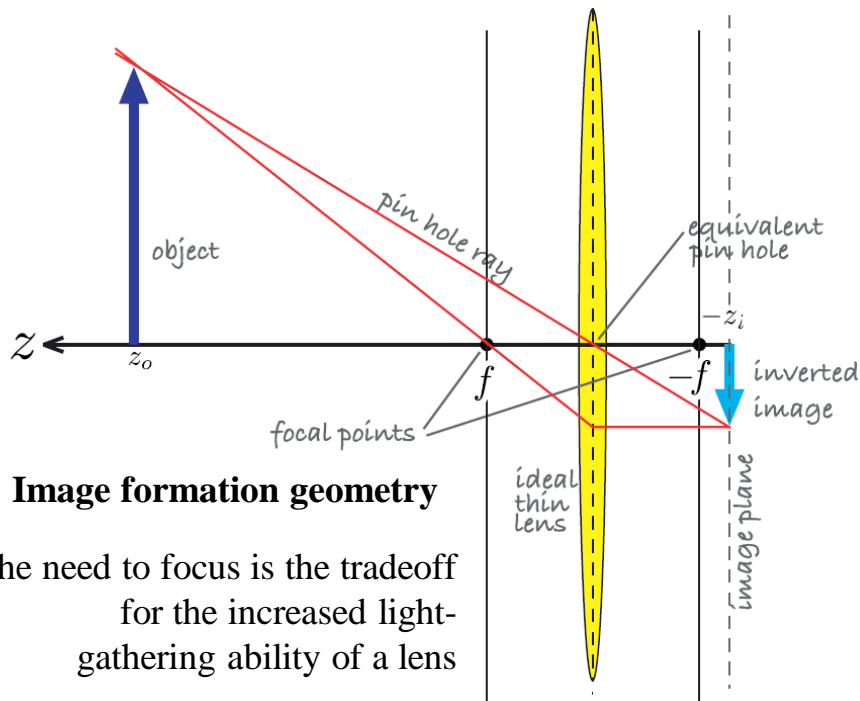
Lens Law $\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$

the distance to the object z_o the distance to the image z_i

the focal length of the lens f

perspective projection $x = f \frac{X}{Z}, y = f \frac{Y}{Z}$

$$P = (X, Y, Z) \xrightarrow{f} p = (x, y)$$



Perspective Transform

Camera Models

Lens Law $\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$

the distance to the object z_o

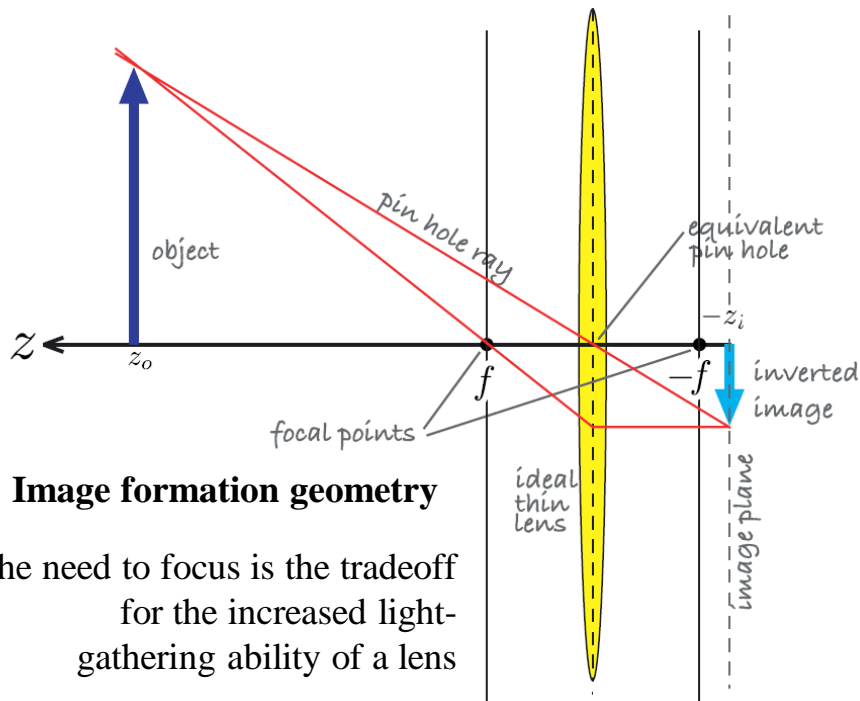
the distance to the image z_i

the focal length of the lens f

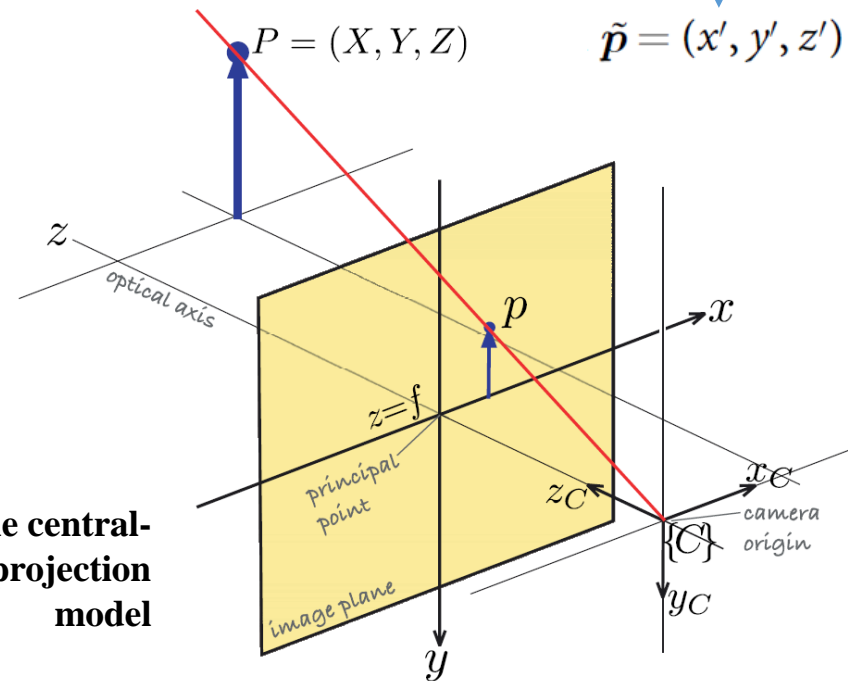
perspective projection $x = f \frac{X}{Z}, y = f \frac{Y}{Z}$

$P = (X, Y, Z) \xrightarrow{f} p = (x, y)$

$\tilde{p} = (x', y', z')$



The central-projection model



Perspective Transform

Camera Models

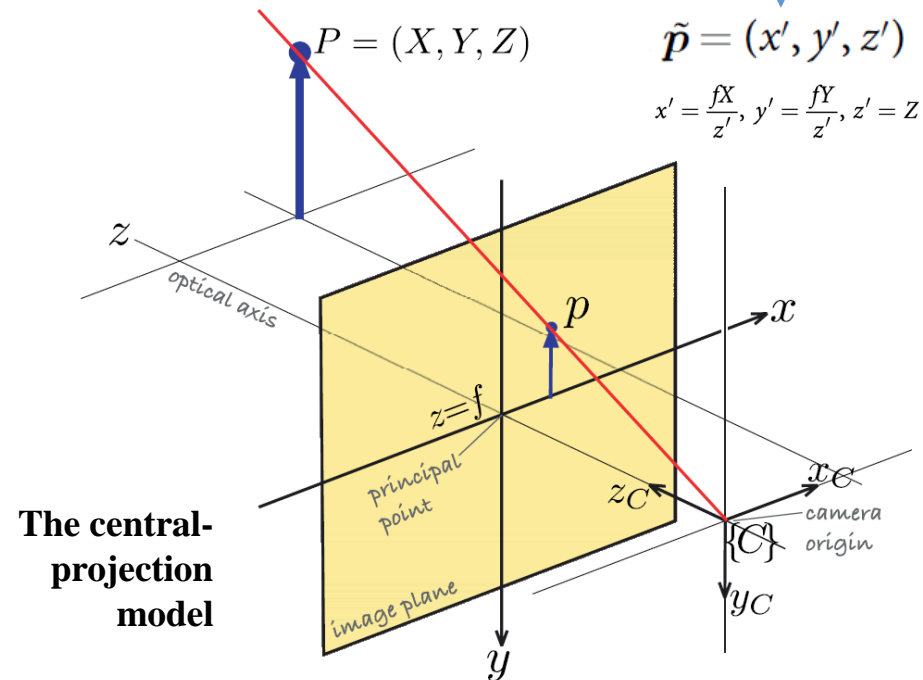
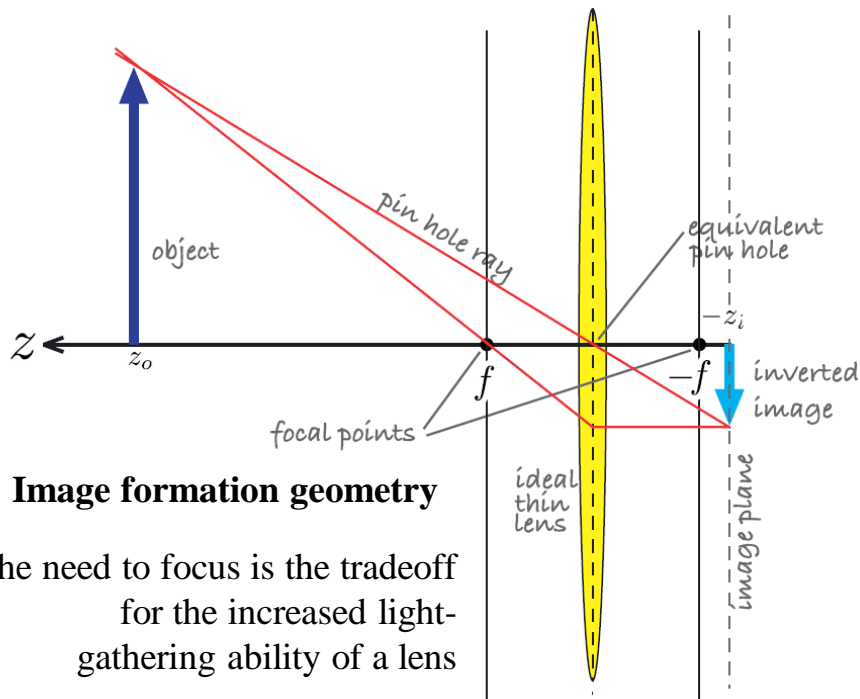
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the distance to the object z_o the distance to the image z_i

the focal length of the lens f

perspective projection $x = f \frac{X}{Z}, y = f \frac{Y}{Z}$

$P = (X, Y, Z) \xrightarrow{f} p = (x, y)$ homogeneous form



Characteristics

Perspective Transform

- A mapping from 3D space to the 2D image plane

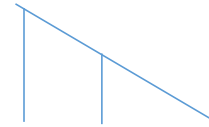
$$\mathbb{R}^3 \mapsto \mathbb{R}^2.$$

Characteristics

Perspective Transform

- A mapping from 3D space to the 2D image plane
- Straight lines in the world are projected to straight lines on the image plane.

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

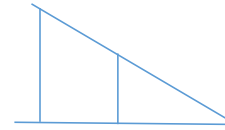


Characteristics

Perspective Transform

- A mapping from 3D space to the 2D image plane
- Straight lines in the world are projected to straight lines on the image plane.
- Parallel lines in the world are projected to lines that intersect at a vanishing point.
 - The exception are lines in the plane parallel to the image plane which do not converge.

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

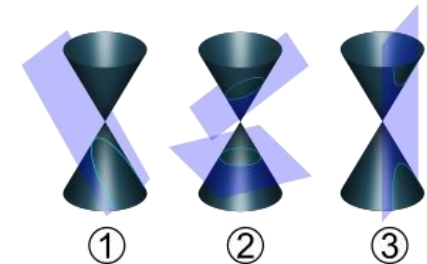
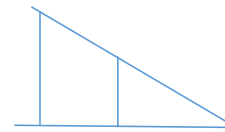


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$$\mathbb{R}^3 \mapsto \mathbb{R}^2.$$

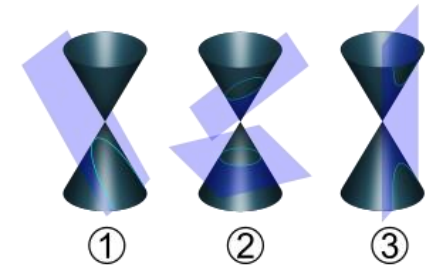
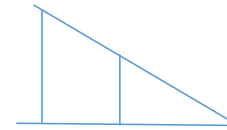


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 - The exception are lines in the plane parallel to the image plane which do not converge.
- Conics in the world are projected to conics on the image plane.
- **The mapping is not one-to-one and a unique inverse does not exist.**

$$\mathbb{R}^3 \mapsto \mathbb{R}^2.$$



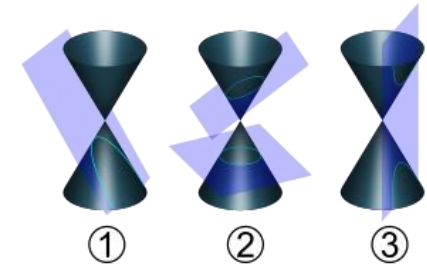
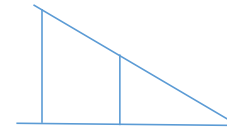
$$P = (X, Y, Z) \xrightarrow{f} p = (x, y)$$

Characteristics

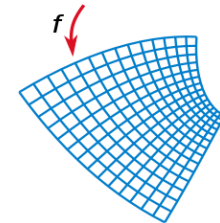
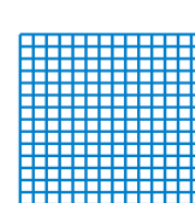
Perspective Transform

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 - The exception are lines in the plane parallel to the image plane which do not converge.
- Conics in the world are projected to conics on the image plane.
- **The mapping is not one-to-one and a unique inverse does not exist.**
- The transformation is not conformal
 - It does not preserve shape since internal angles are not preserved, different from translation, rotation and scaling.

$$\mathbb{R}^3 \mapsto \mathbb{R}^2.$$



$$P = (X, Y, Z) \xrightarrow{f} p = (x, y)$$



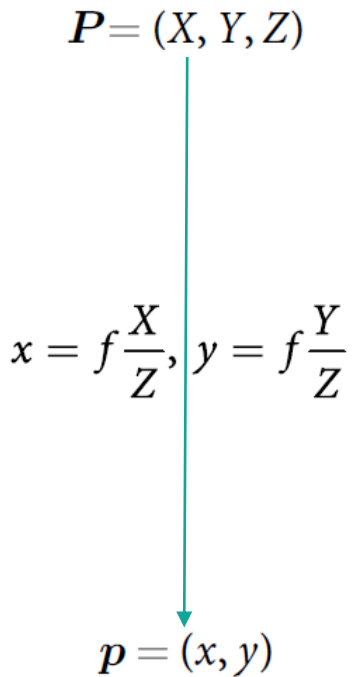
Retinal Image Plane Coordinates

Written in homogeneous form

$$P = (X, Y, Z)$$

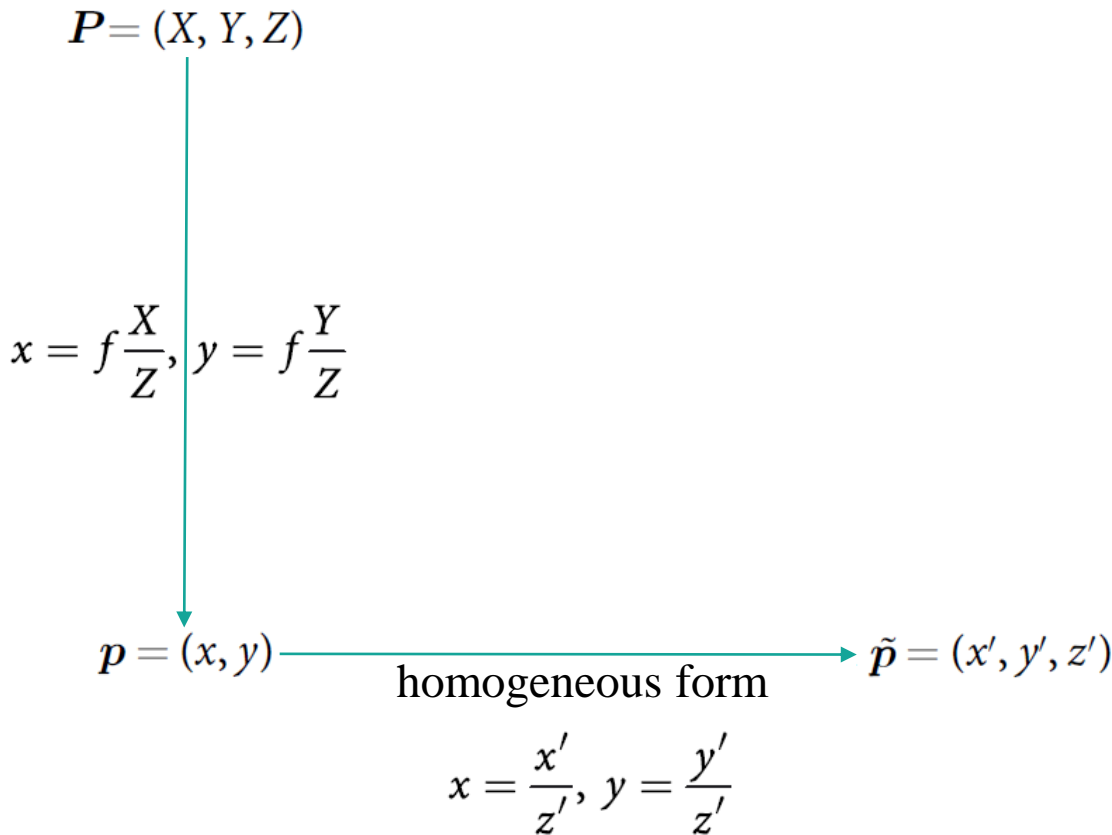
Retinal Image Plane Coordinates

Written in homogeneous form



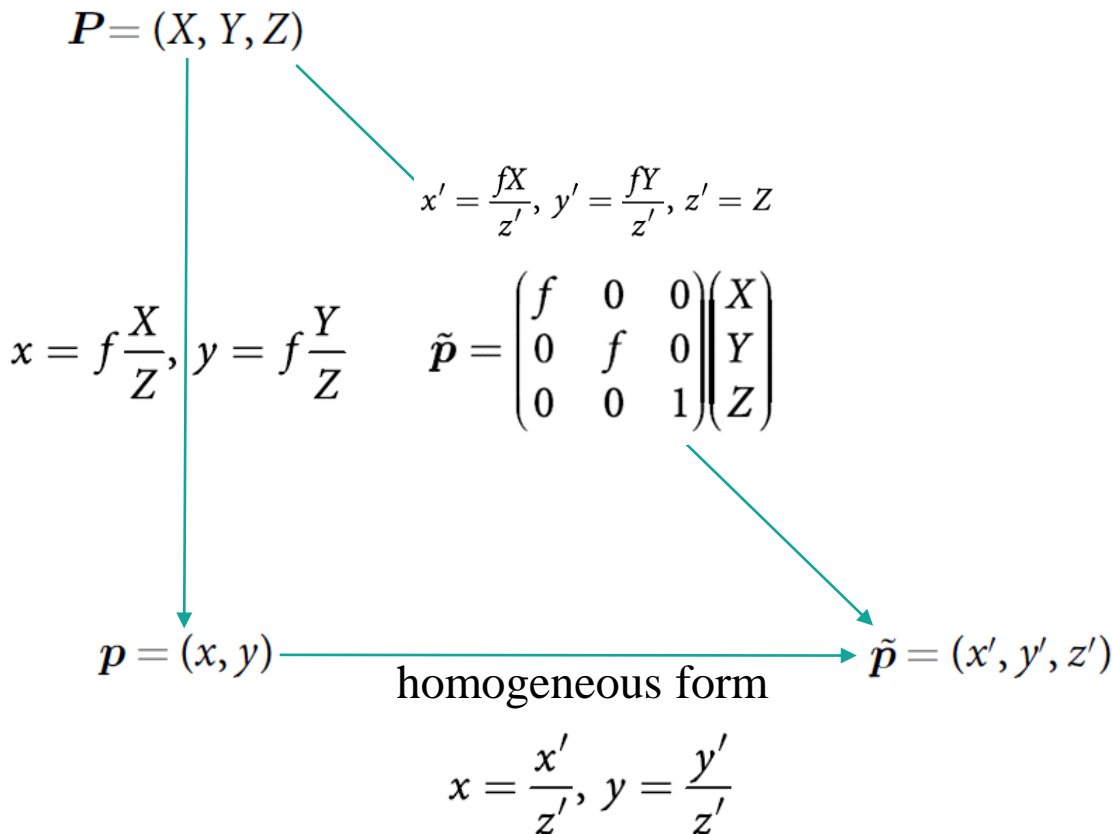
Retinal Image Plane Coordinates

Written in homogeneous form



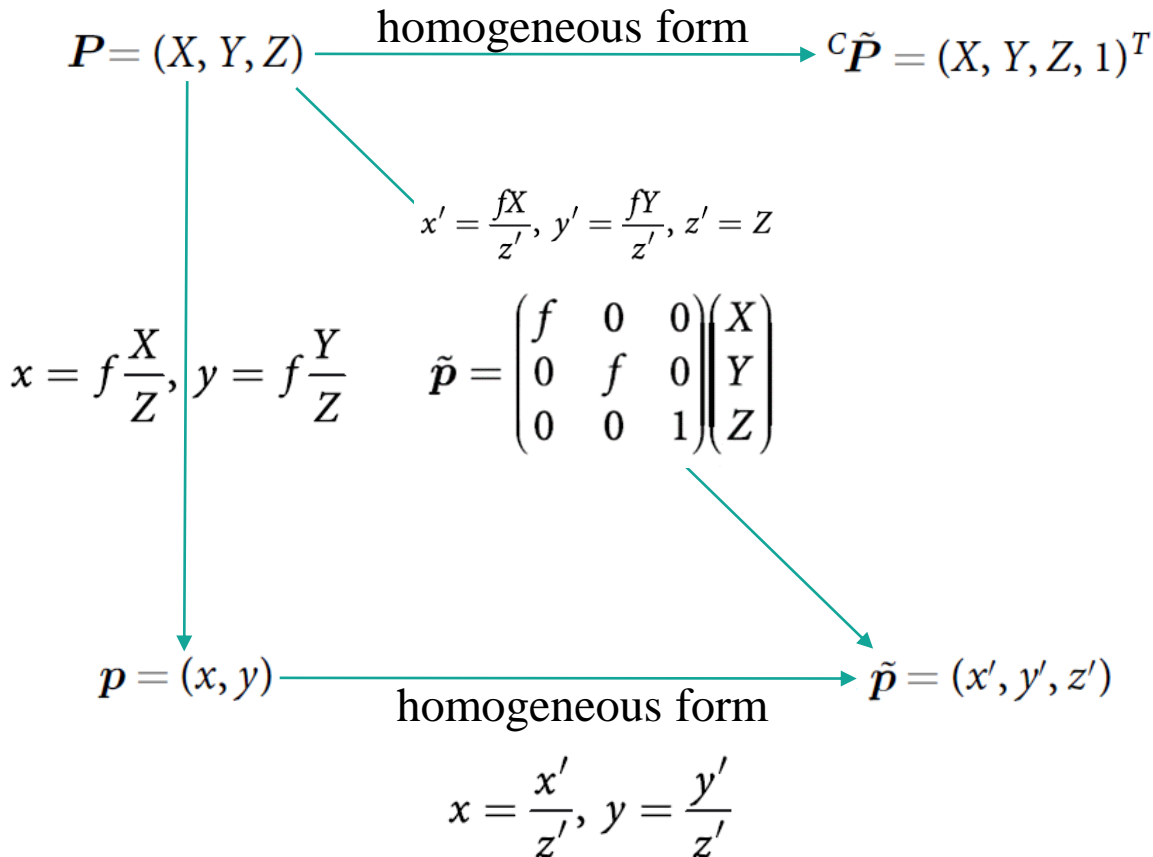
Retinal Image Plane Coordinates

Written in homogeneous form



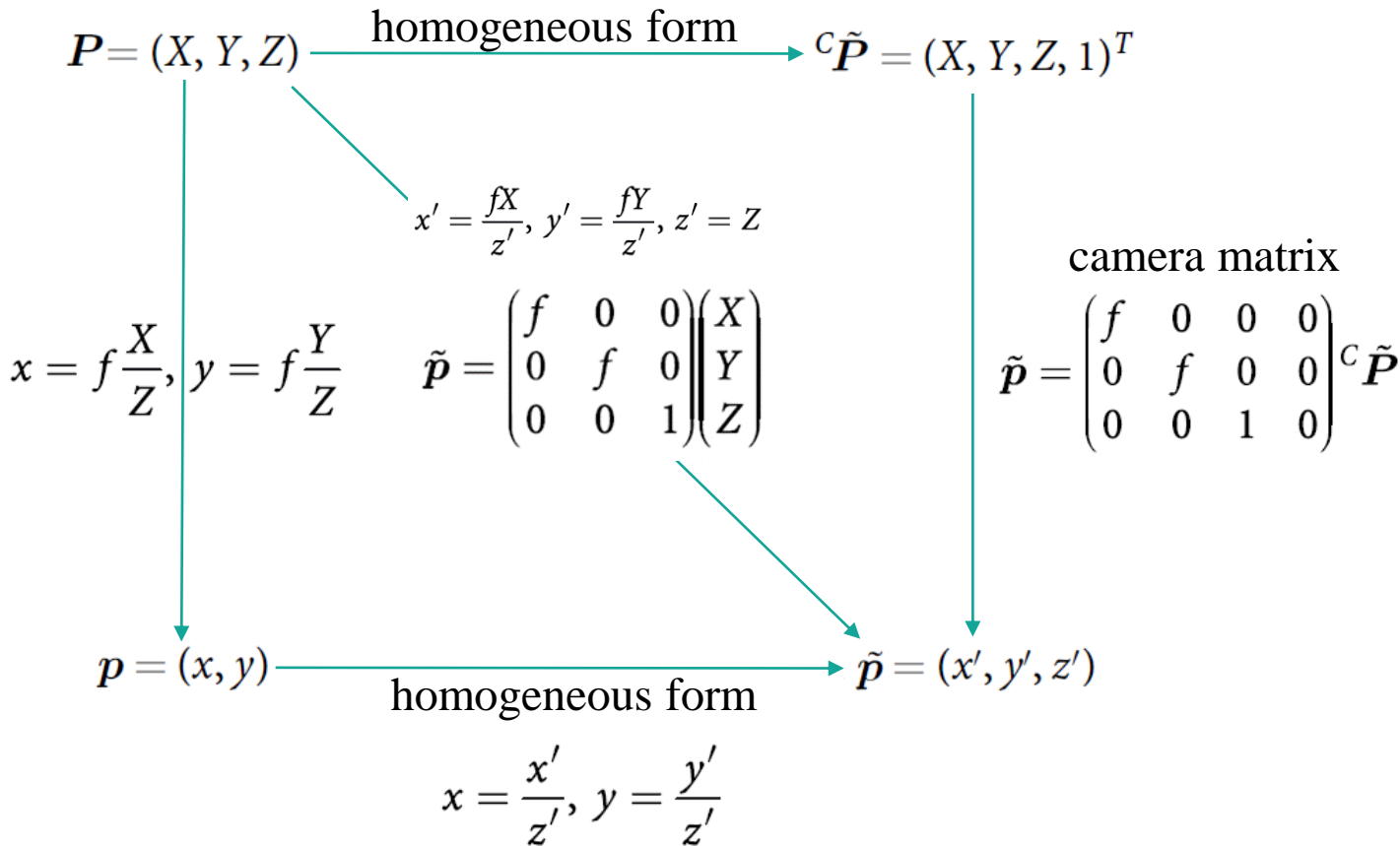
Retinal Image Plane Coordinates

Written in homogeneous form



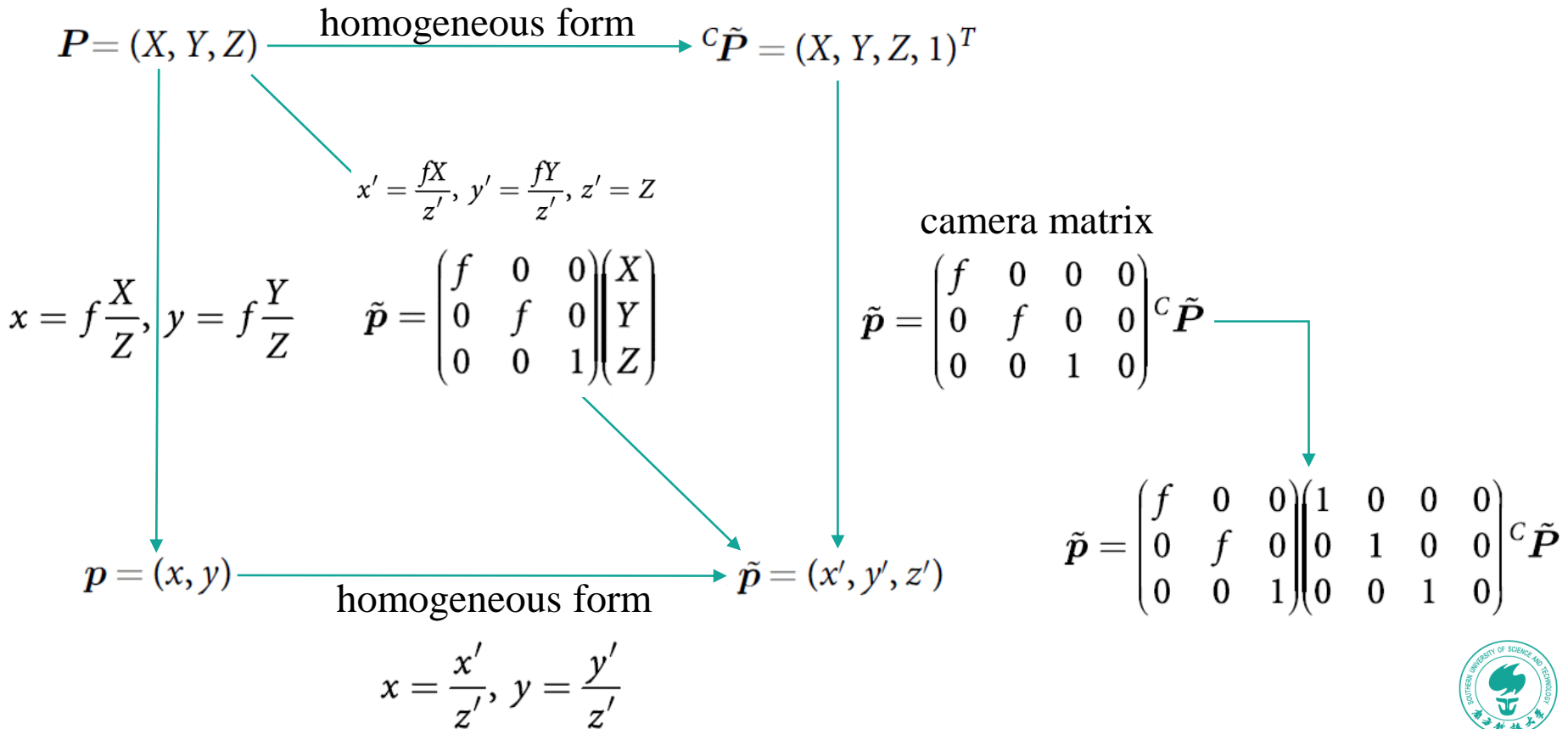
Retinal Image Plane Coordinates

Written in homogeneous form



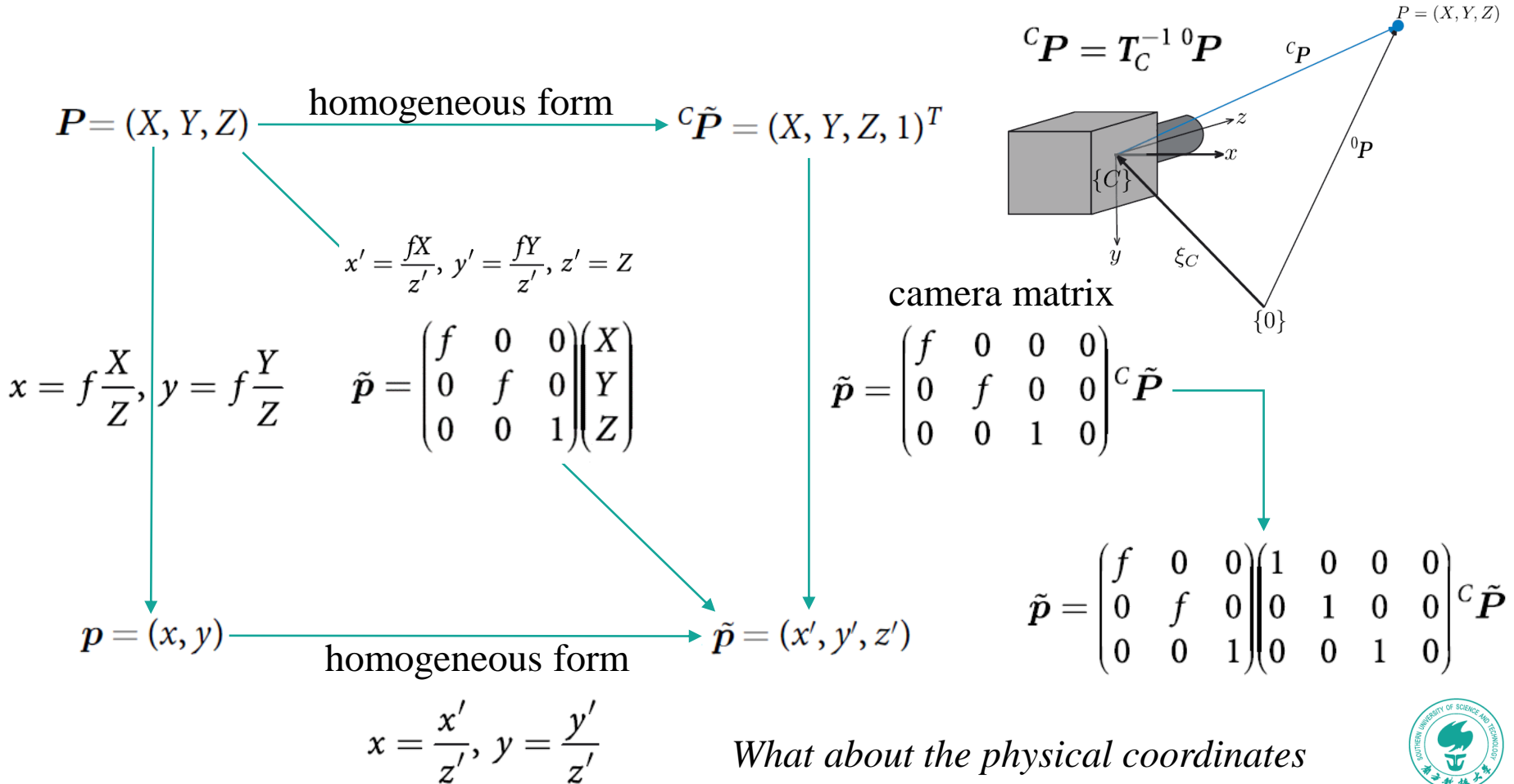
Retinal Image Plane Coordinates

Written in homogeneous form



Retinal Image Plane Coordinates

Written in homogeneous form

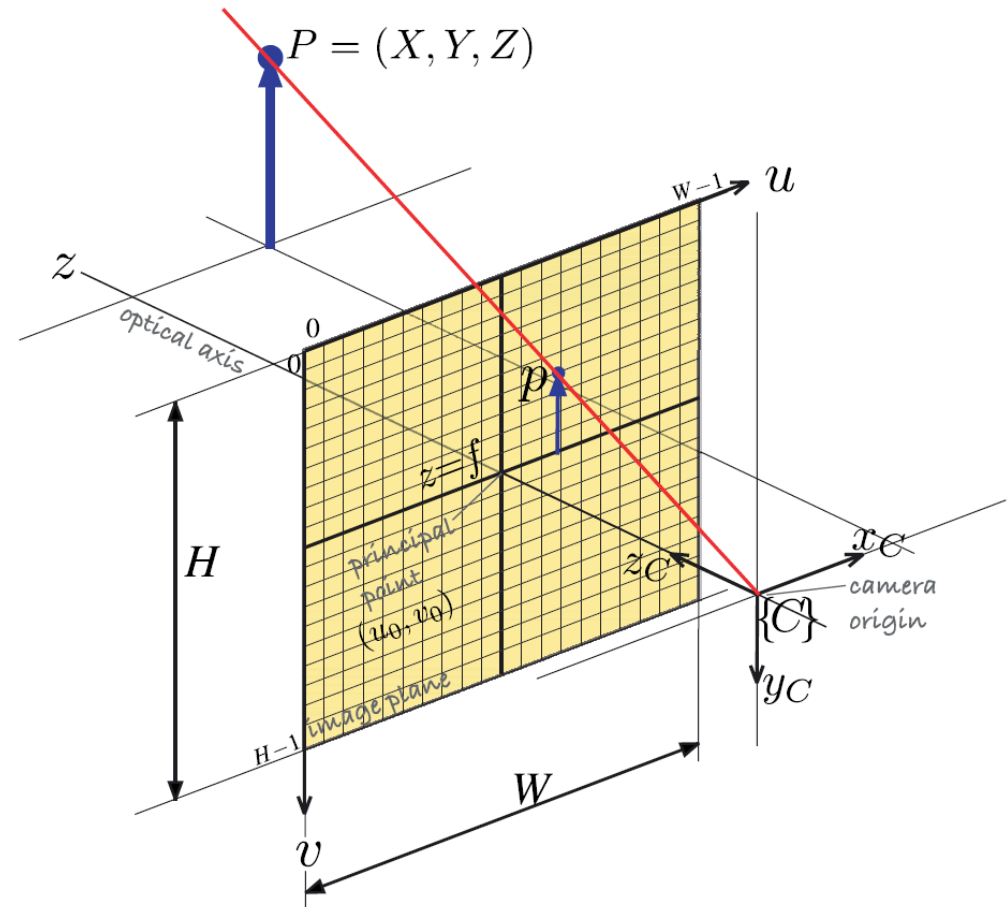


What about the physical coordinates on the actual image?

Express w.r.t the Camera

Physical Meanings of Camera Pixels

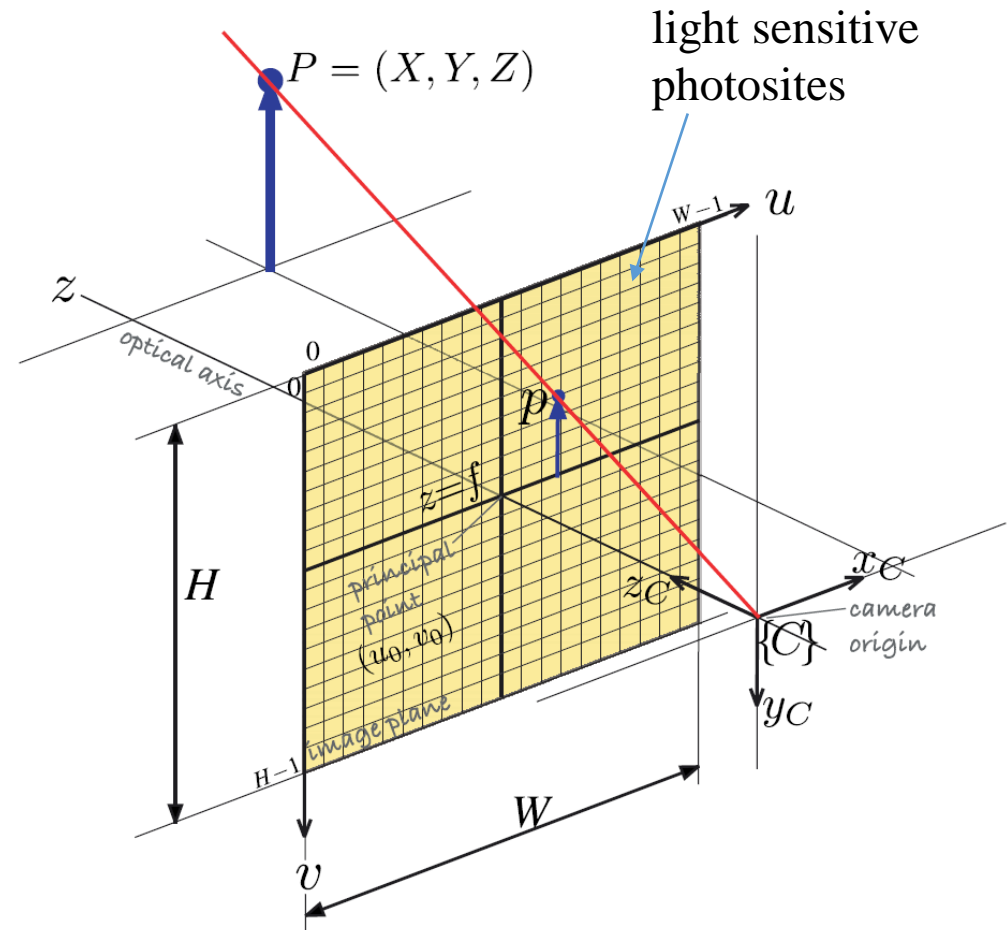
- A **camera sensor** with a $W \times H$ grid of image pixels
 - The pixel coordinates (u, v)



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels
 - The pixel coordinates (u, v)

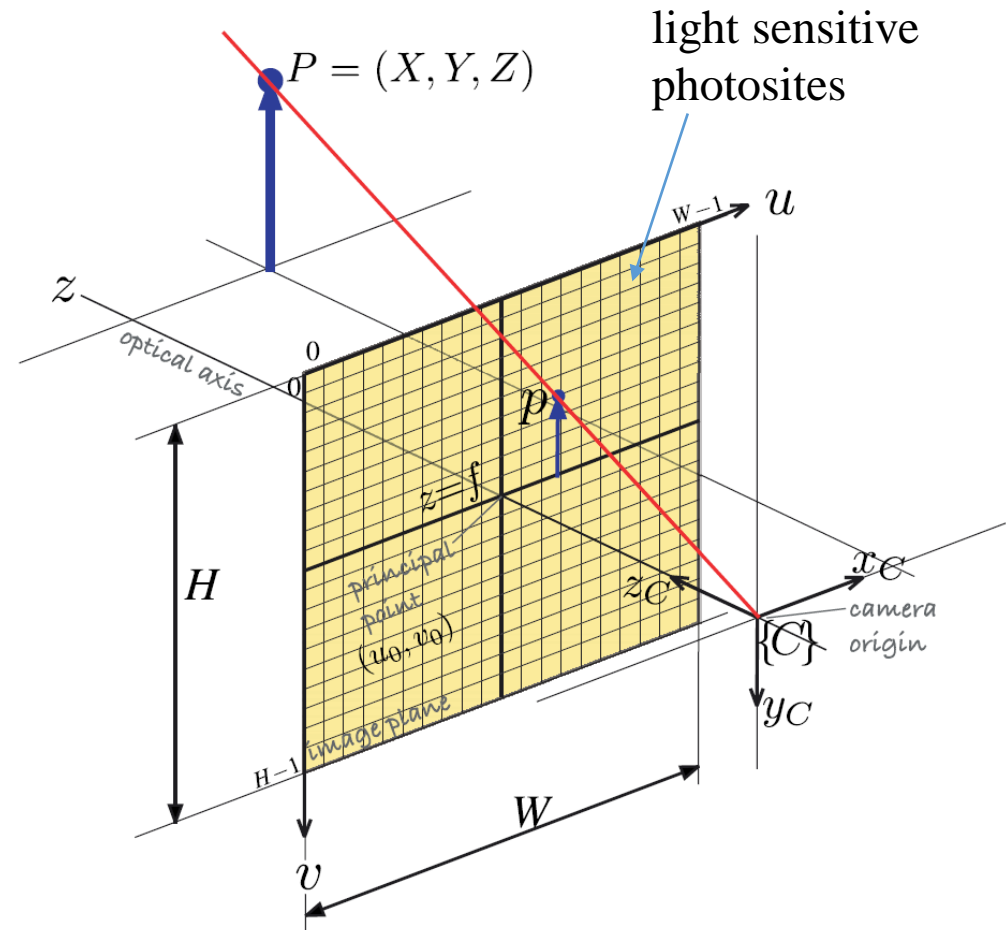


Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels
 - The pixel coordinates (u, v)

$$u = \frac{x}{\rho_w} + u_0, v = \frac{y}{\rho_h} + v_0$$



Express w.r.t the Camera

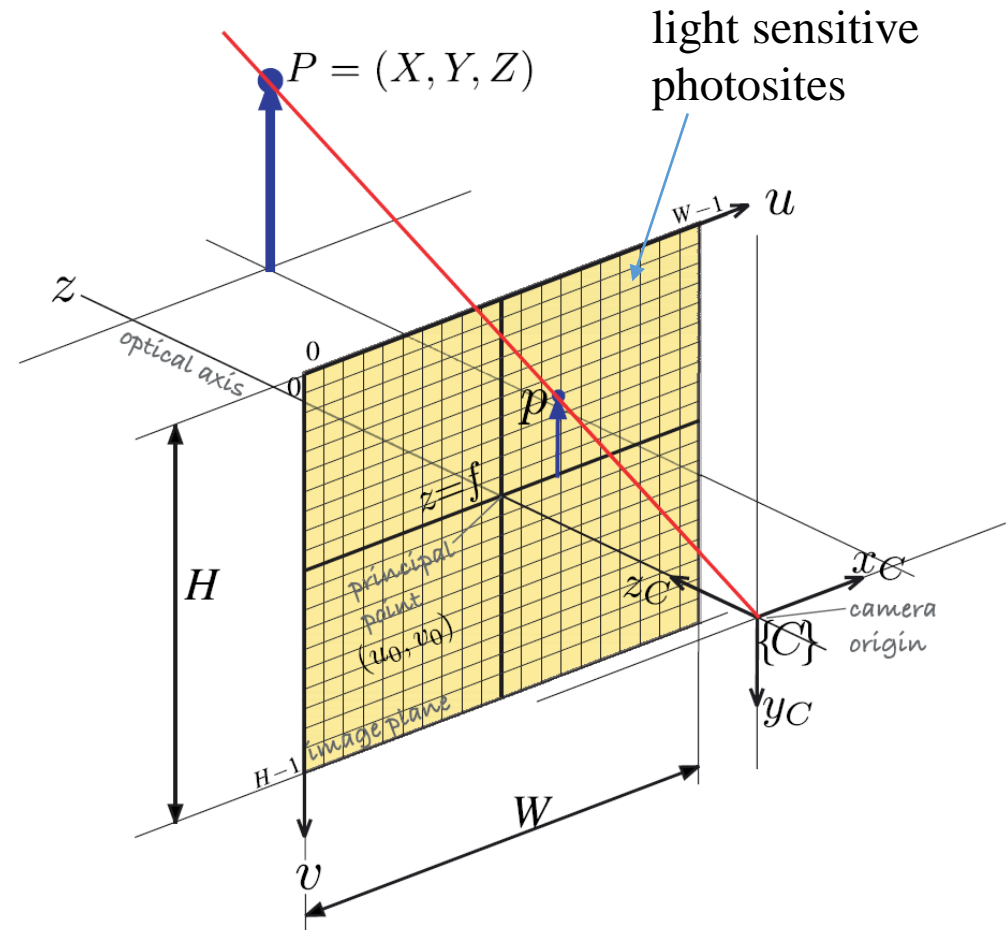
Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

- The pixel coordinates (u, v)

Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$



Express w.r.t the Camera

Physical Meanings of Camera Pixels

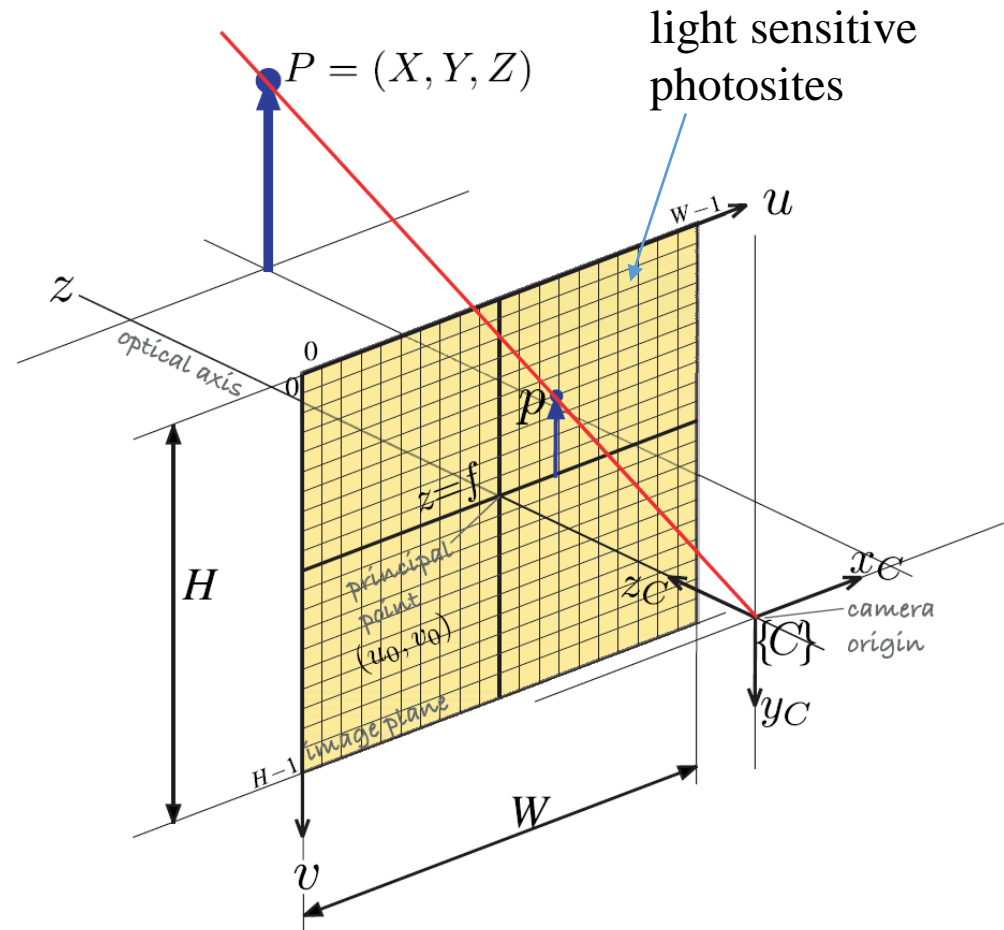
- A **camera sensor** with a $\mathbf{W} \times \mathbf{H}$ grid of image pixels

- The pixel coordinates (u, v)

Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

width and height of each pixel



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

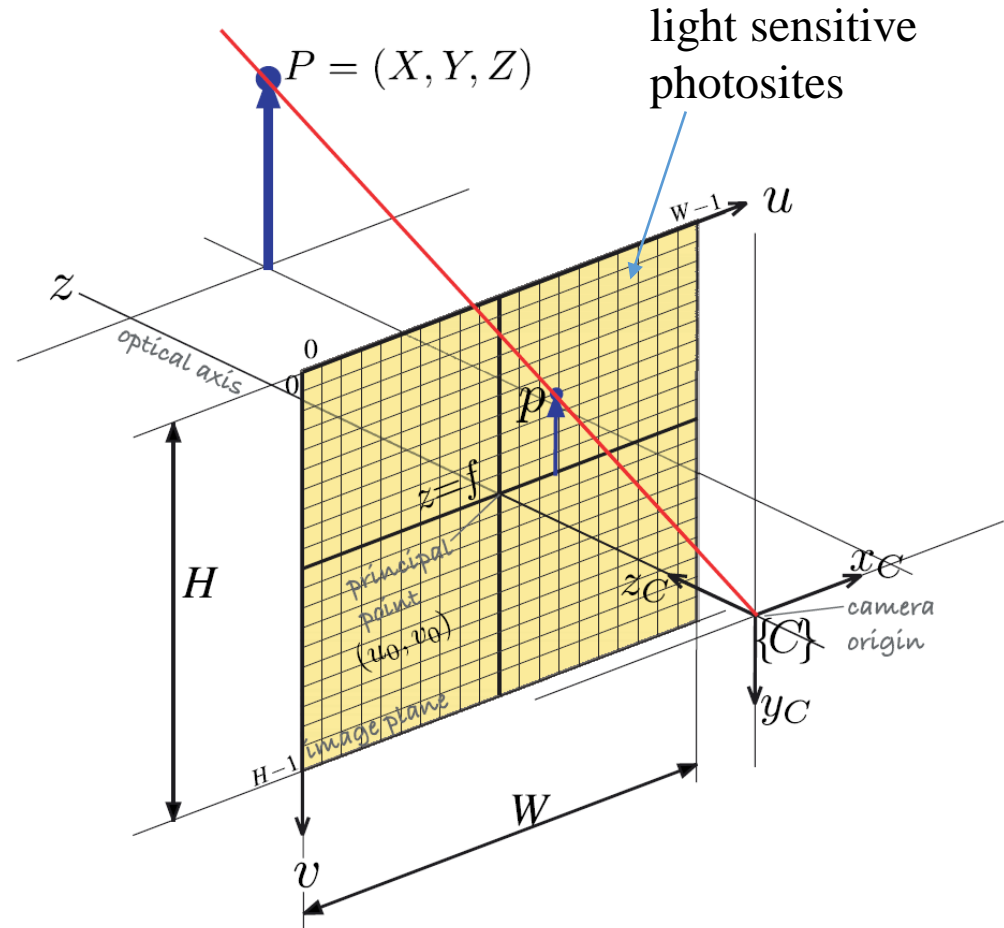
- The pixel coordinates (u, v)

Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

width and height of each pixel

$\tilde{\mathbf{p}} = (u', v', w')$
pixel coordinate



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

- The pixel coordinates (u, v)

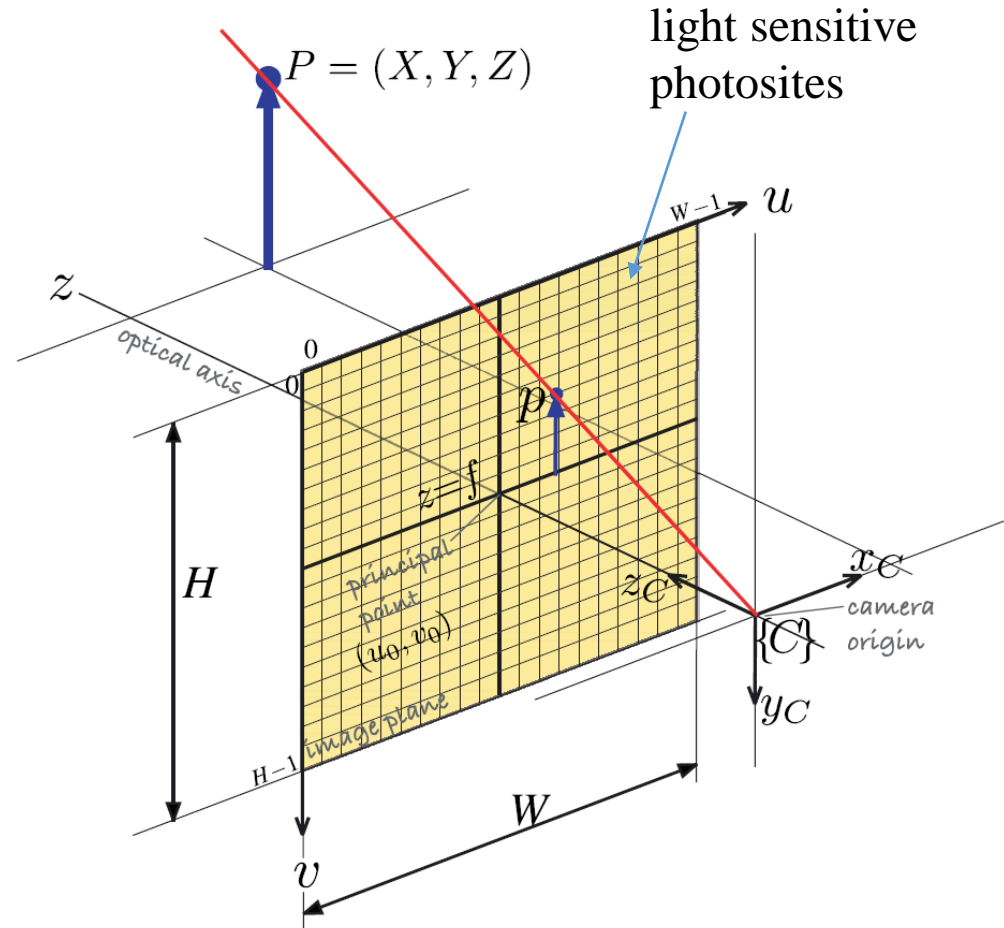
Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

width and height of each pixel

$\tilde{p} = (u', v', w')$
pixel coordinate

$$u = \frac{u'}{w'}, \quad v = \frac{v'}{w'}$$



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

- The pixel coordinates (u, v)

Principal point in pixel coordinate

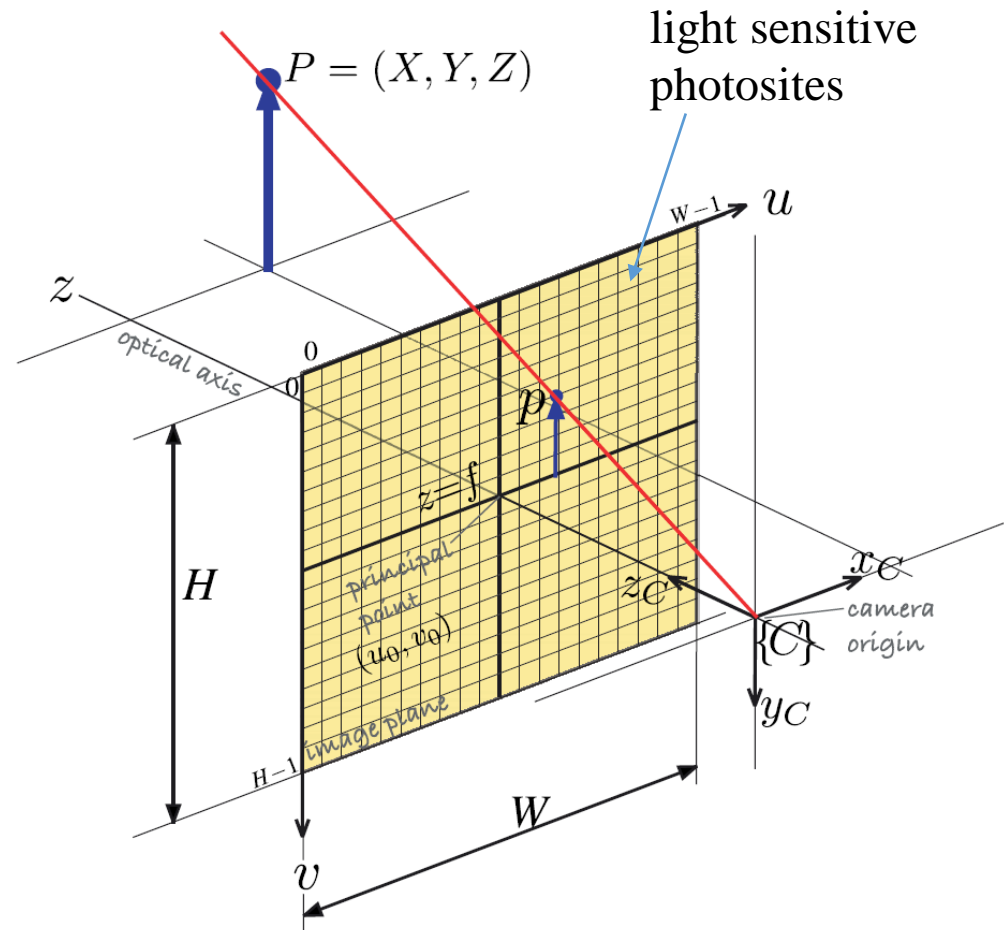
$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

width and height of each pixel

$\tilde{\mathbf{p}} = (u', v', w')$
pixel coordinate

$$u = \frac{u'}{w'}, \quad v = \frac{v'}{w'}$$

$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} 1/\rho_w & 0 & u_0 \\ 0 & 1/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} {}^c\tilde{\mathbf{P}}$$



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

- The pixel coordinates (u, v)

Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

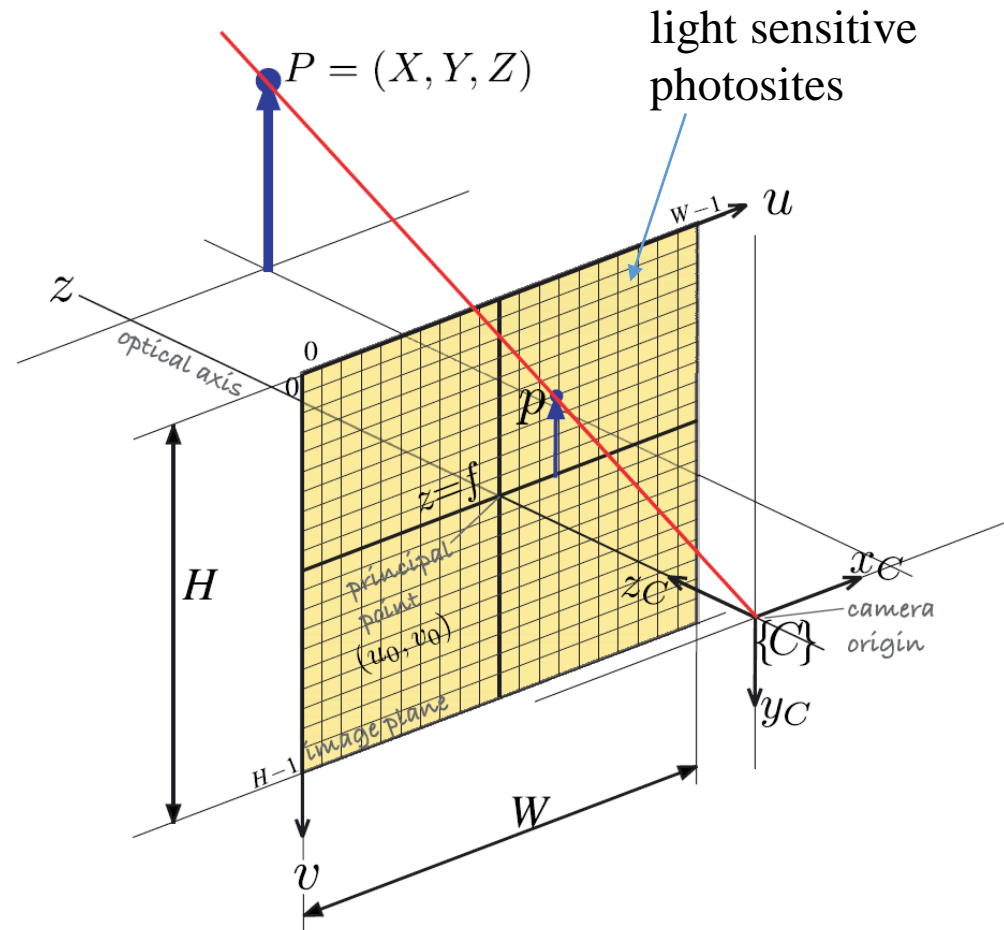
width and height of each pixel

$\tilde{p} = (u', v', w')$
pixel coordinate

$$u = \frac{u'}{w'}, \quad v = \frac{v'}{w'}$$

$$\tilde{p} = \underbrace{\begin{pmatrix} 1/\rho_w & 0 & u_0 \\ 0 & 1/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_K \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} {}^c \tilde{P}$$

camera parameter matrix



Express w.r.t the Camera

Physical Meanings of Camera Pixels

- A **camera sensor** with a $W \times H$ grid of image pixels

- The pixel coordinates (u, v)

Principal point in pixel coordinate

$$u = \frac{x}{\rho_w} + u_0, \quad v = \frac{y}{\rho_h} + v_0$$

width and height of each pixel

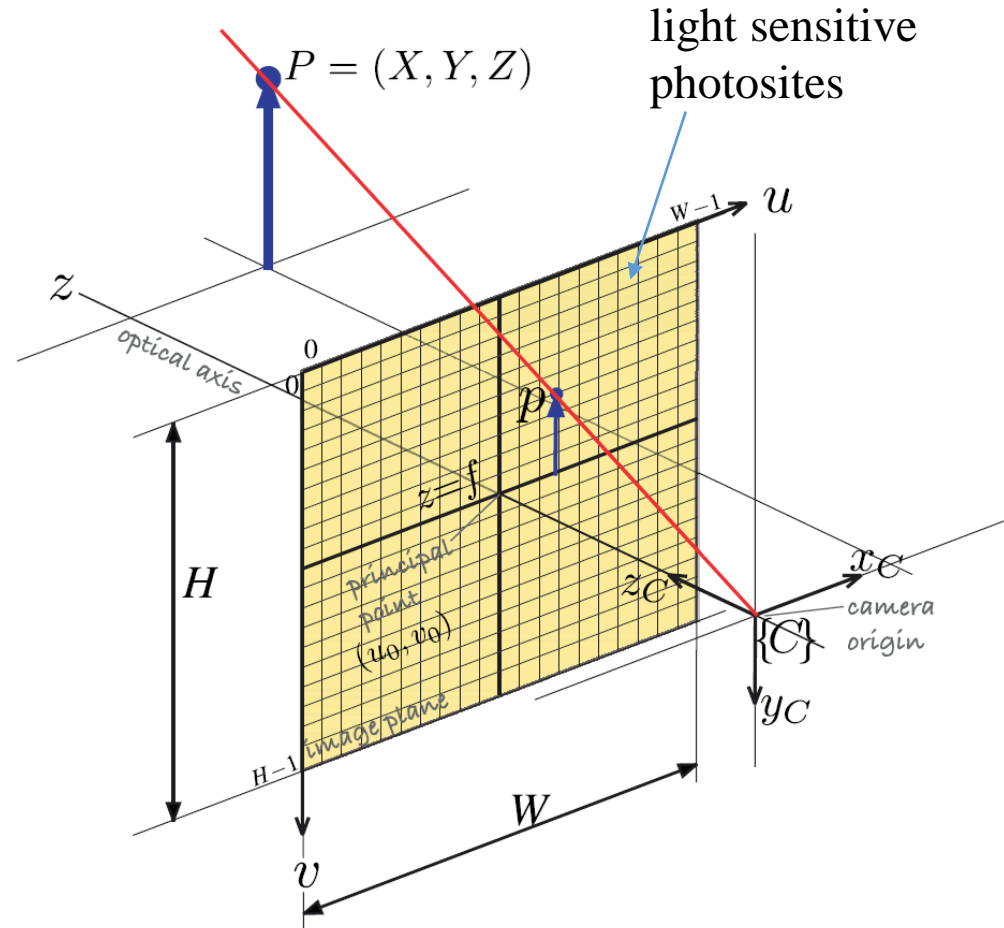
$\tilde{p} = (u', v', w')$
pixel coordinate

$$u = \frac{u'}{w'}, \quad v = \frac{v'}{w'}$$

camera (projection) matrix

$$\tilde{p} = \underbrace{\begin{pmatrix} 1/\rho_w & 0 & u_0 \\ 0 & 1/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_K \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} {}^c \tilde{P}$$

camera parameter matrix



Camera Projection In General Form

Still, something is missing

- The 3x4 Camera Calibration Matrix
 - Performs scaling, translation and perspective projection

$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{\mathbf{P}}$$
$$= \mathbf{K} \mathbf{P}_0 {}^0T_C^{-1} \tilde{\mathbf{P}}$$
$$= \mathbf{C} \tilde{\mathbf{P}}$$

Camera Projection In General Form

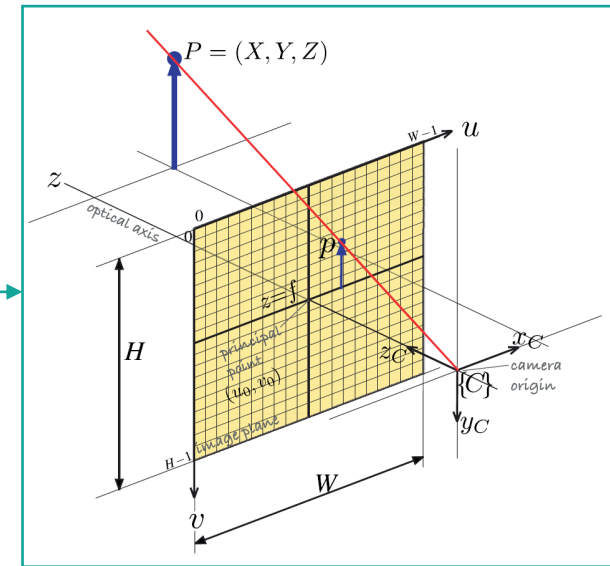
Still, something is missing

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$$\tilde{p} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}} \tilde{P}$$

$$= K P_0 {}^0T_C^{-1} \tilde{P}$$

$$= C \tilde{P}$$



Camera Projection In General Form

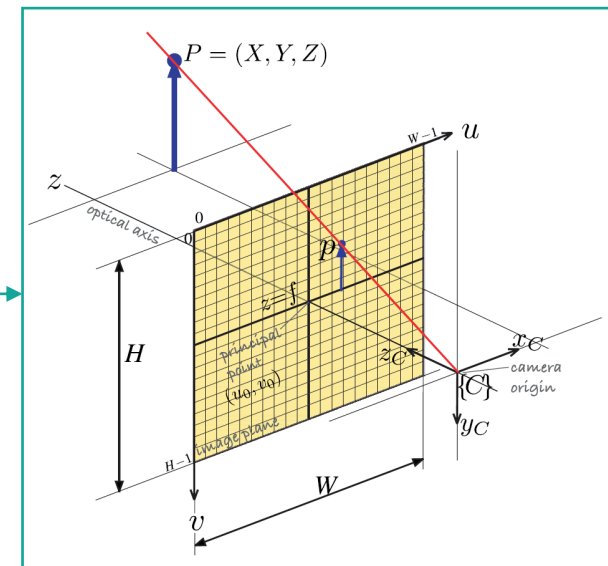
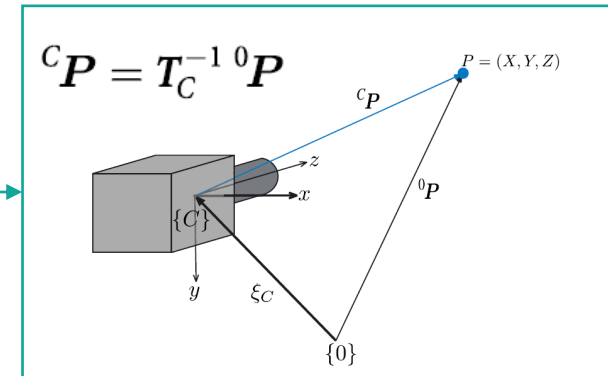
Still, something is missing

- The 3x4 Camera Calibration Matrix
 - Performs scaling, translation and perspective projection

$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{\mathbf{P}}$$

$$= \mathbf{K} \mathbf{P}_0 {}^0T_C^{-1} \tilde{\mathbf{P}}$$

$$= \mathbf{C} \tilde{\mathbf{P}}$$



Camera Projection In General Form

Still, something is missing

- The 3x4 Camera Calibration Matrix
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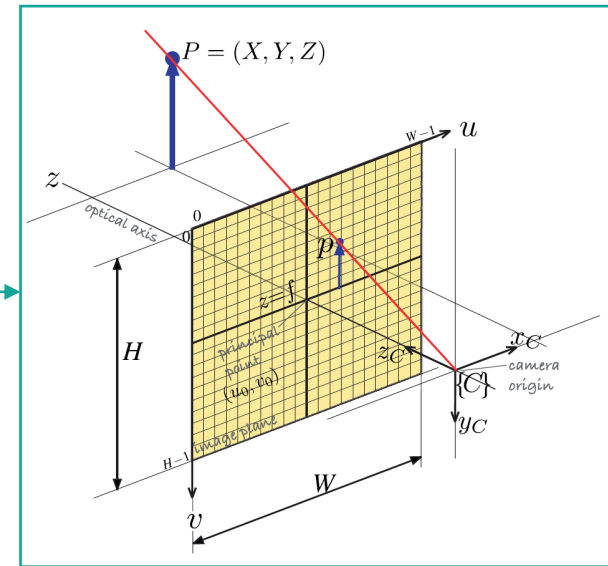
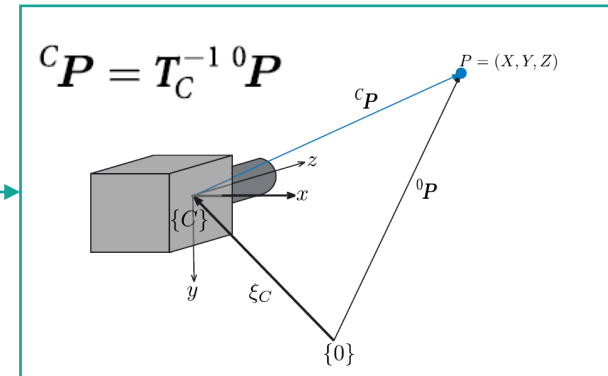
$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{\mathbf{P}}$$

$$= \mathbf{K} \mathbf{P}_0 {}^0T_C^{-1} \tilde{\mathbf{P}}$$

$$= \mathbf{C} \tilde{\mathbf{P}}$$

↓

$$\mathbf{p} = \mathcal{P}(\mathbf{P}, \mathbf{K}, \xi_C)$$



Camera Projection In General Form

Still, something is missing

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 - Performs scaling, translation and perspective projection

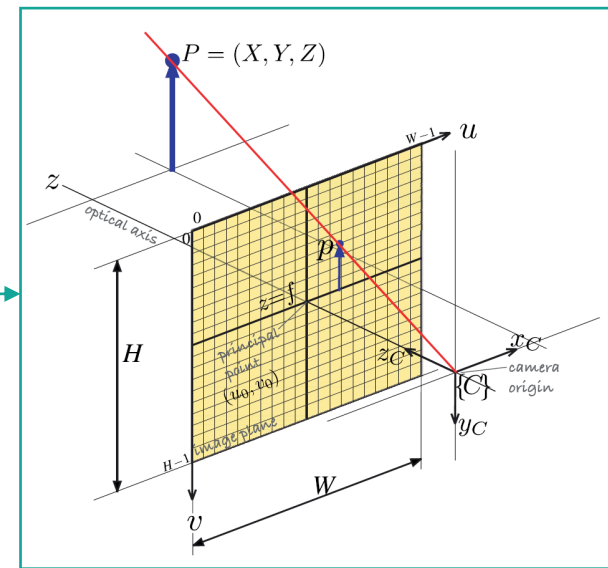
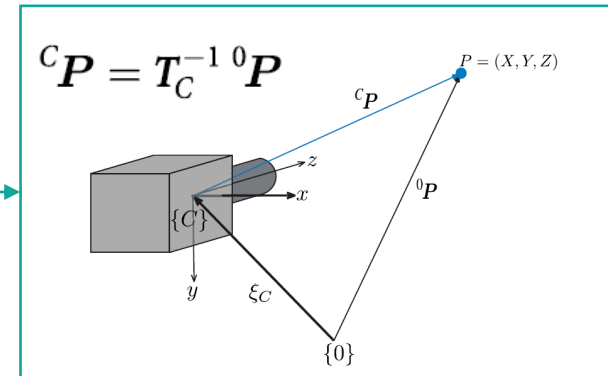
$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{\mathbf{P}}$$

$$= \mathbf{K} \mathbf{P}_0 {}^0T_C^{-1} \tilde{\mathbf{P}}$$

$$= \mathbf{C} \tilde{\mathbf{P}}$$

↓

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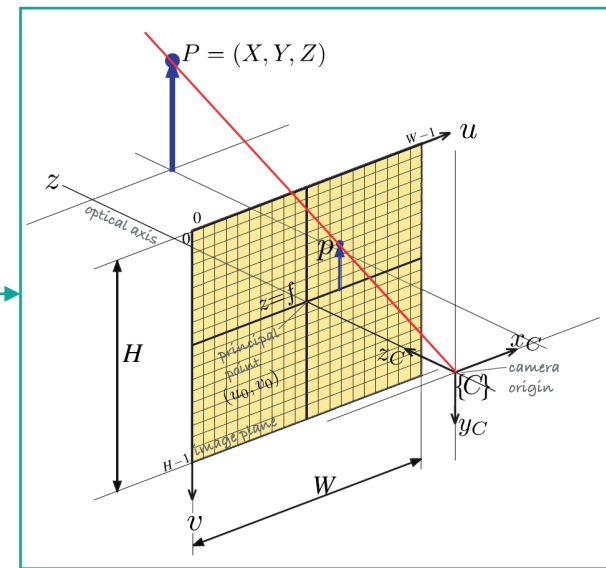
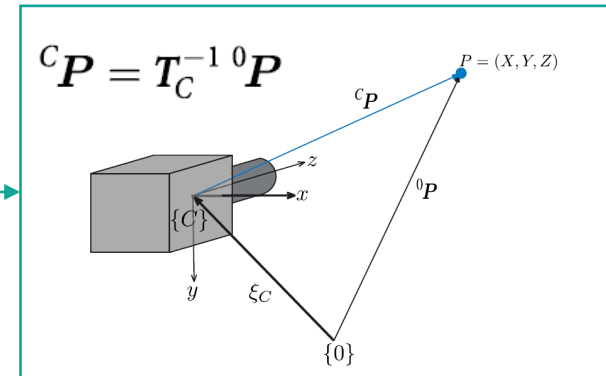
$$= K P_0 {}^0T_C^{-1} \tilde{P}$$

$$= C \tilde{P}$$



$$p = \mathcal{P}(P, K, \xi_C)$$

Camera parameter matrix: 5 parameters



Camera Projection In General Form

Still, something is missing

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$$\tilde{\mathbf{p}} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{\mathbf{P}}$$

$$= \mathbf{K} \mathbf{P}_0 {}^0T_C^{-1} \tilde{\mathbf{P}}$$

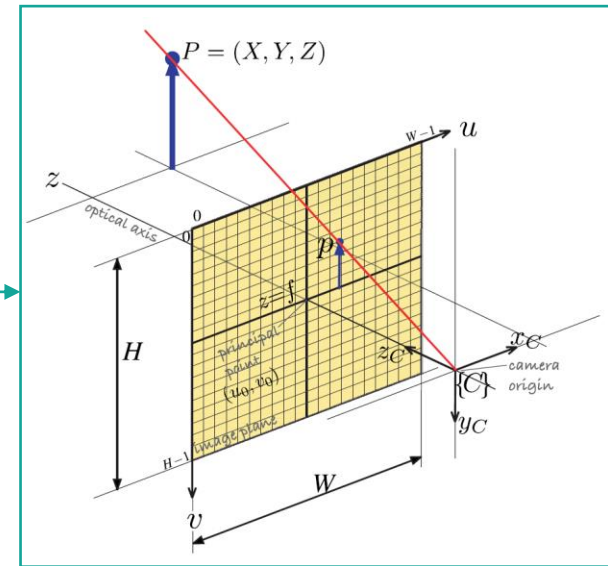
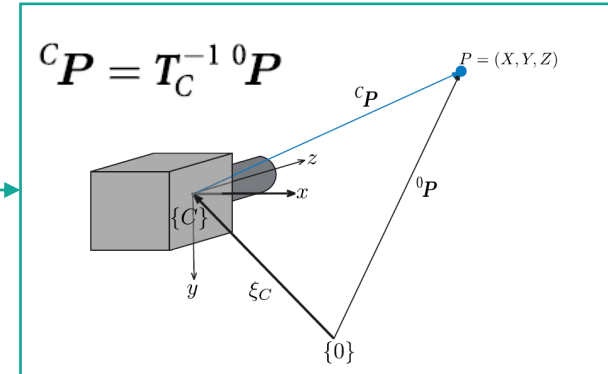
$$= \mathbf{C} \tilde{\mathbf{P}}$$



$$\mathbf{p} = \mathcal{P}(\mathbf{P}, \mathbf{K}, \xi_C)$$

Camera pose: 6 parameters

Camera parameter matrix: 5 parameters



Camera Projection In General Form

Still, something is missing

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 - Performs scaling, translation and perspective projection

$$\tilde{p} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{P}$$

$$= K P_0 {}^0T_C^{-1} \tilde{P}$$

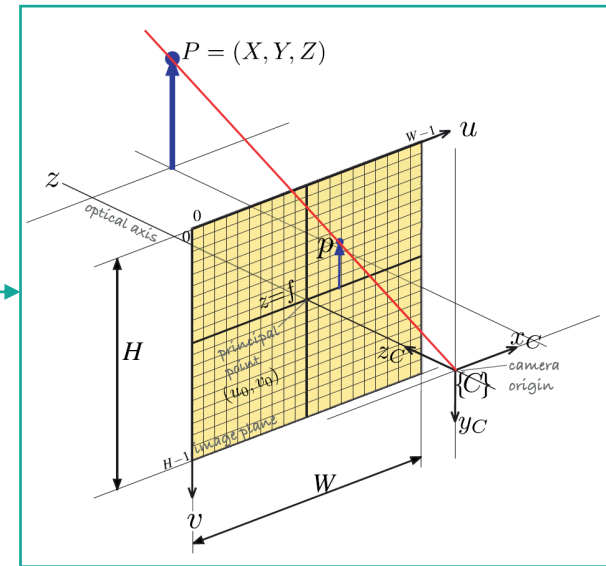
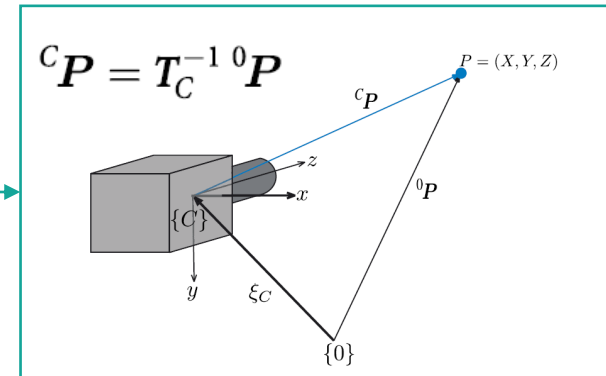
$$= C \tilde{P}$$

C is 3x4 with 12 elements



$p = \mathcal{P}(P, K, \xi_C)$

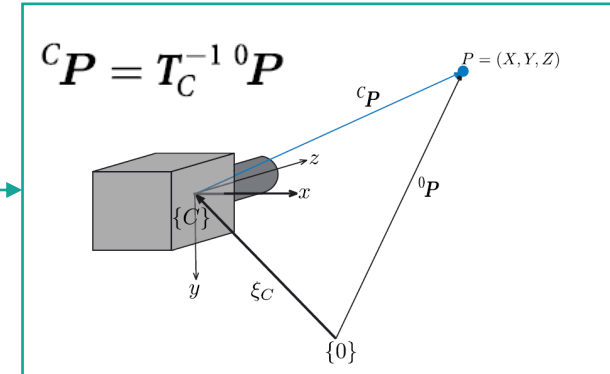
- Camera pose: 6 parameters
- Camera parameter matrix: 5 parameters



Camera Projection In General Form

Still, something is missing

- The 3x4 Camera Calibration Matrix
 - Performs scaling, translation and perspective projection



$$\tilde{p} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{P}$$

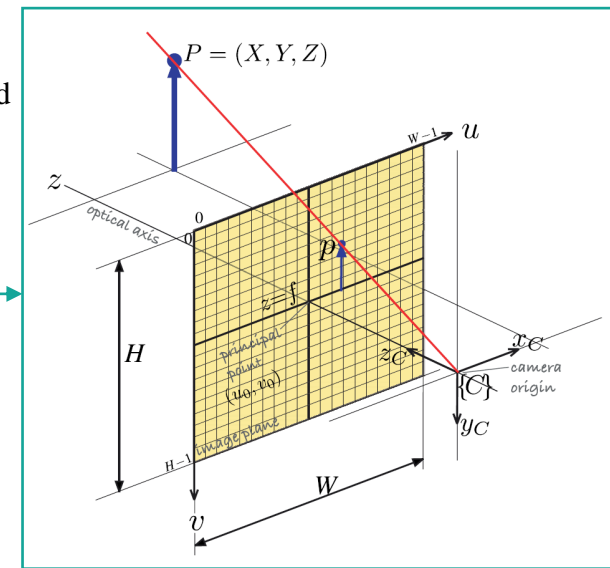
$$= KP_0 {}^0T_C^{-1} \tilde{P}$$

$$= C\tilde{P}$$

$$\tilde{p} = (CH^{-1})(H\tilde{P}) = C'\tilde{P}'$$

C is 3x4 with 12 elements

Unconstrained Overall Scale Factor



$$p = \mathcal{P}(P, K, \xi_C)$$

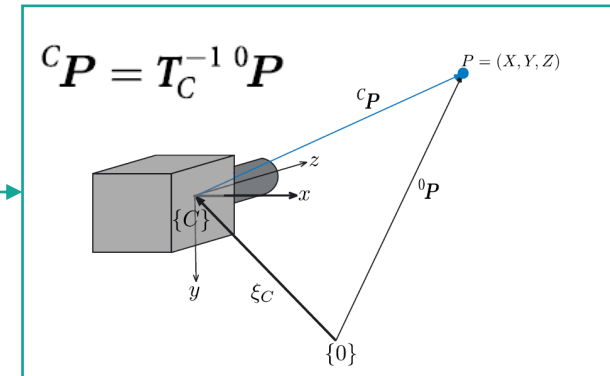
Camera pose: 6 parameters

Camera parameter matrix: 5 parameters

Camera Projection In General Form

Still, something is missing

- The 3x4 Camera Calibration Matrix
 - Performs scaling, translation and perspective projection



$$\tilde{p} = \underbrace{\begin{pmatrix} f/\rho_w & 0 & u_0 \\ 0 & f/\rho_h & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{extrinsic}} \underbrace{({}^0T_C)^{-1}}_{\text{extrinsic}} \tilde{P}$$

$$= KP_0 {}^0T_C^{-1} \tilde{P}$$

$$= C\tilde{P}$$

$$\tilde{p} = (CH^{-1})(H\tilde{P}) = C'\tilde{P}'$$

C is 3x4 with 12 elements

Unconstrained Overall Scale Factor

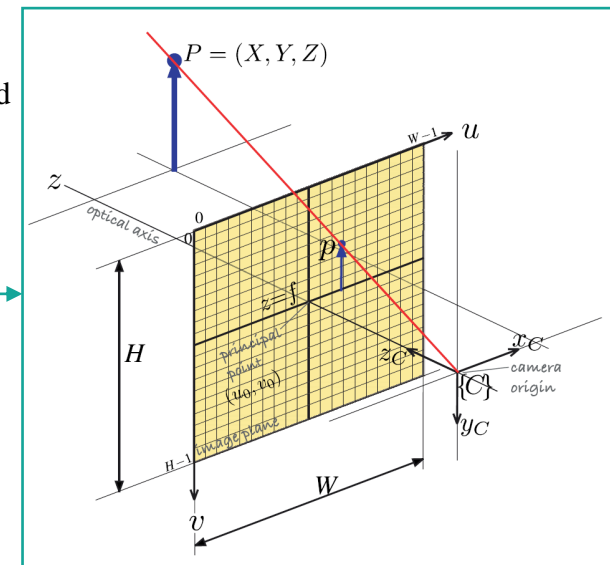


$$p = \mathcal{P}(P, K, \xi_C)$$

Camera pose: 6 parameters

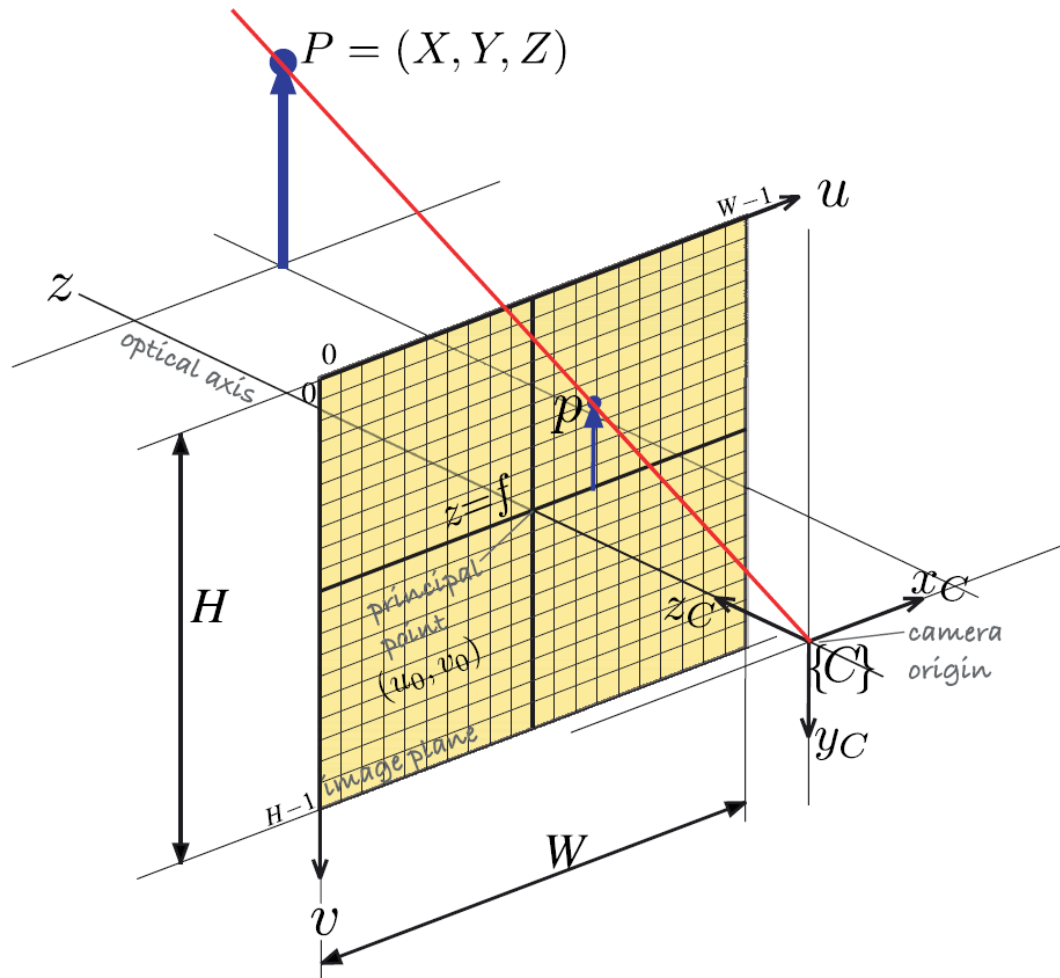
Camera parameter matrix: 5 parameters

It can only be solved if we have information about the camera or the 3D object



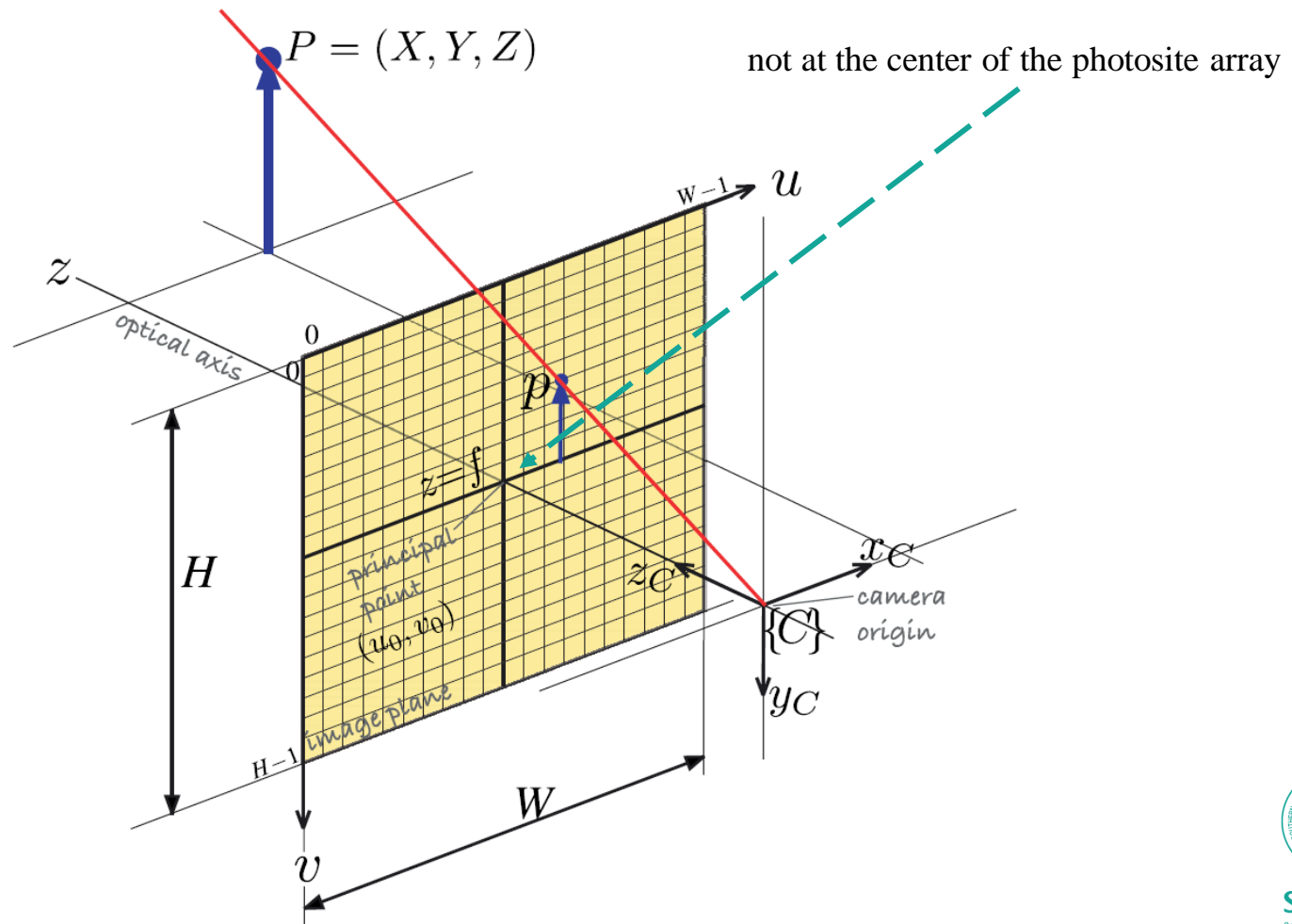
Camera Calibration

In general, the cameras are not made as modeled



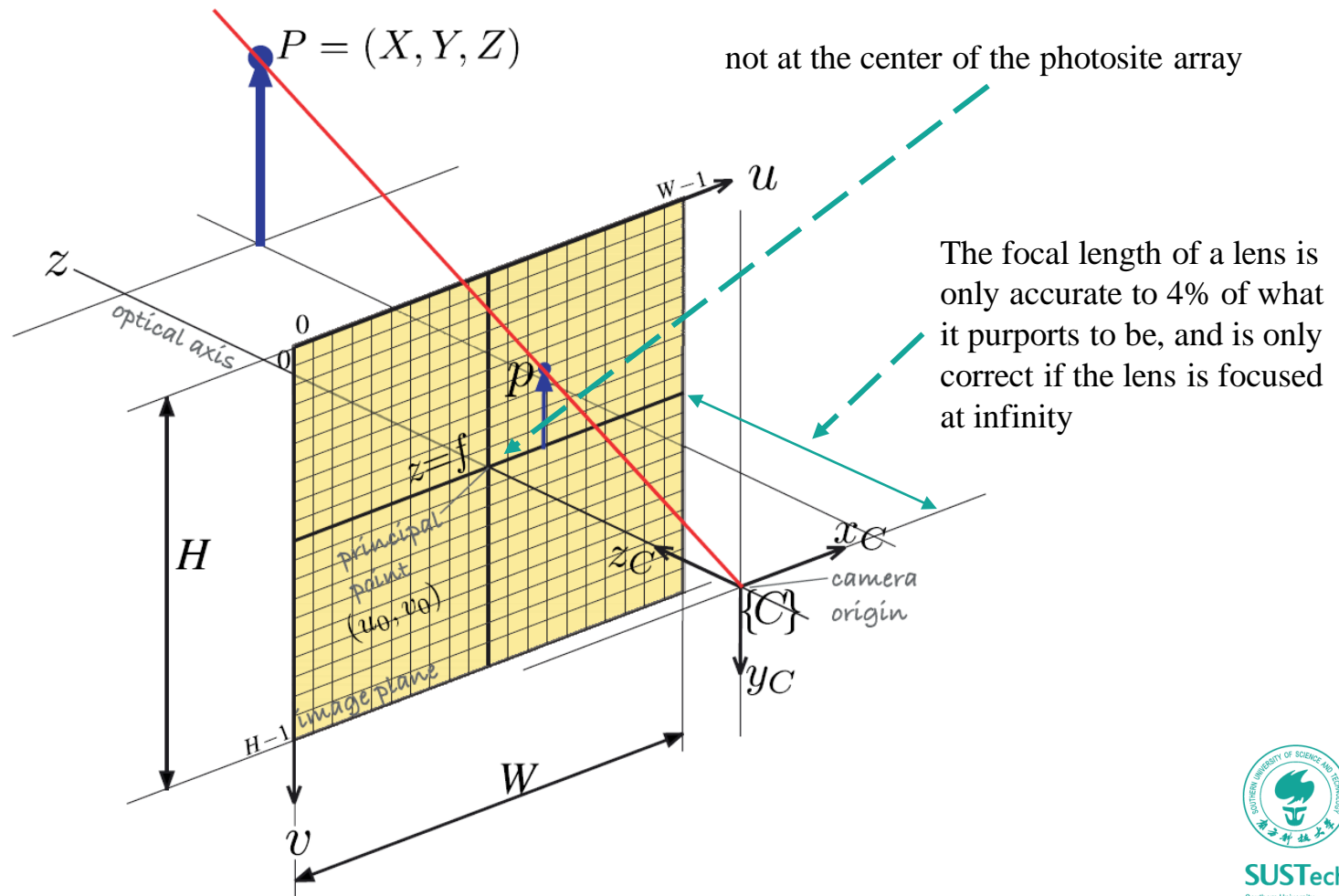
Camera Calibration

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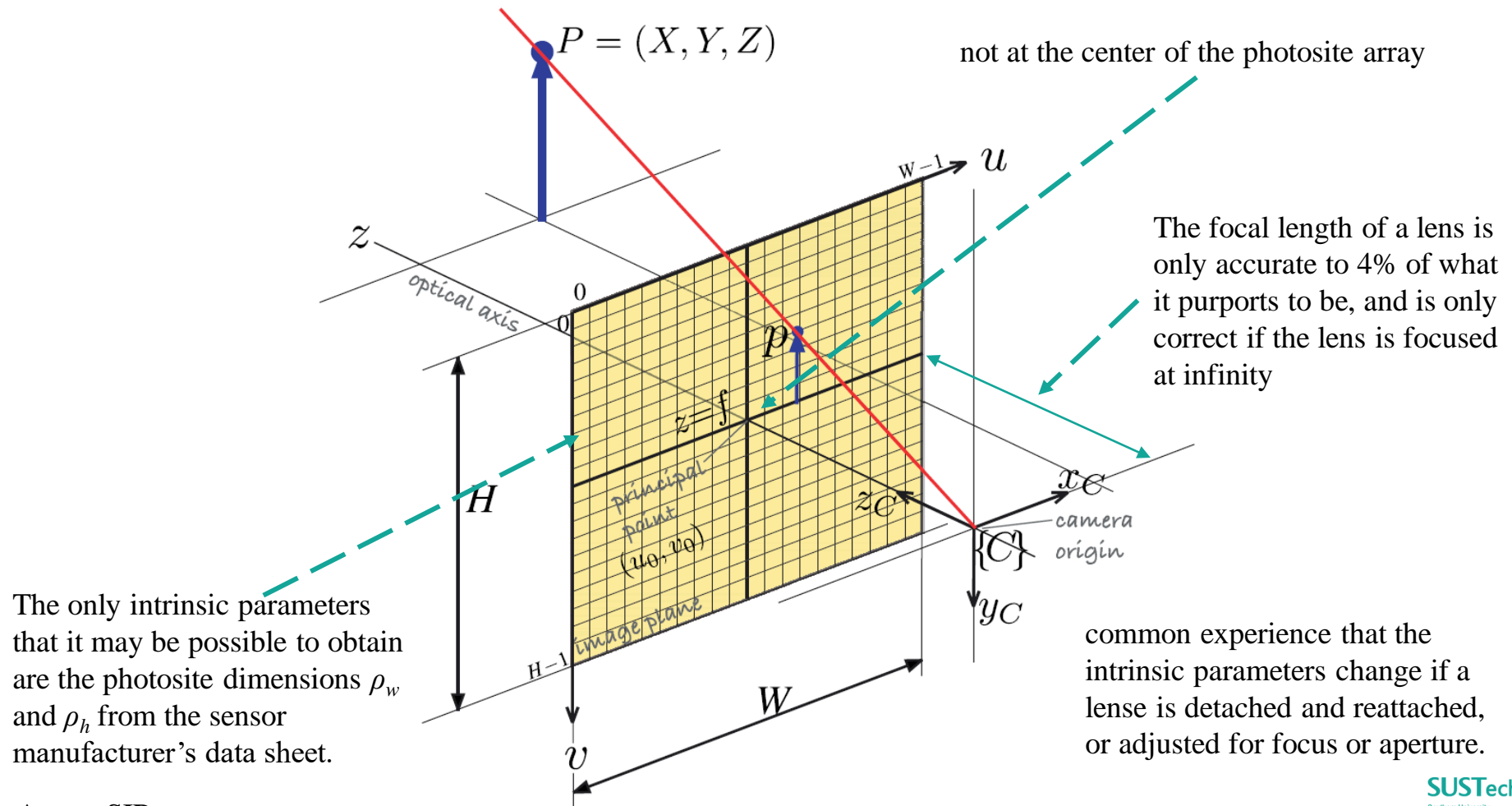
Camera Calibration

In general, the cameras are not made as modeled



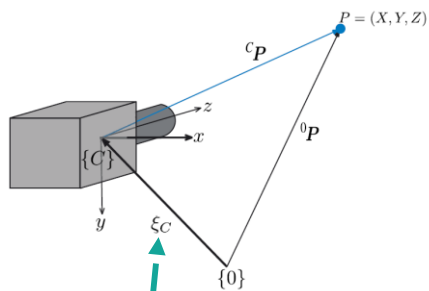
Camera Calibration

In general, the cameras are not made as modeled



Camera Calibration

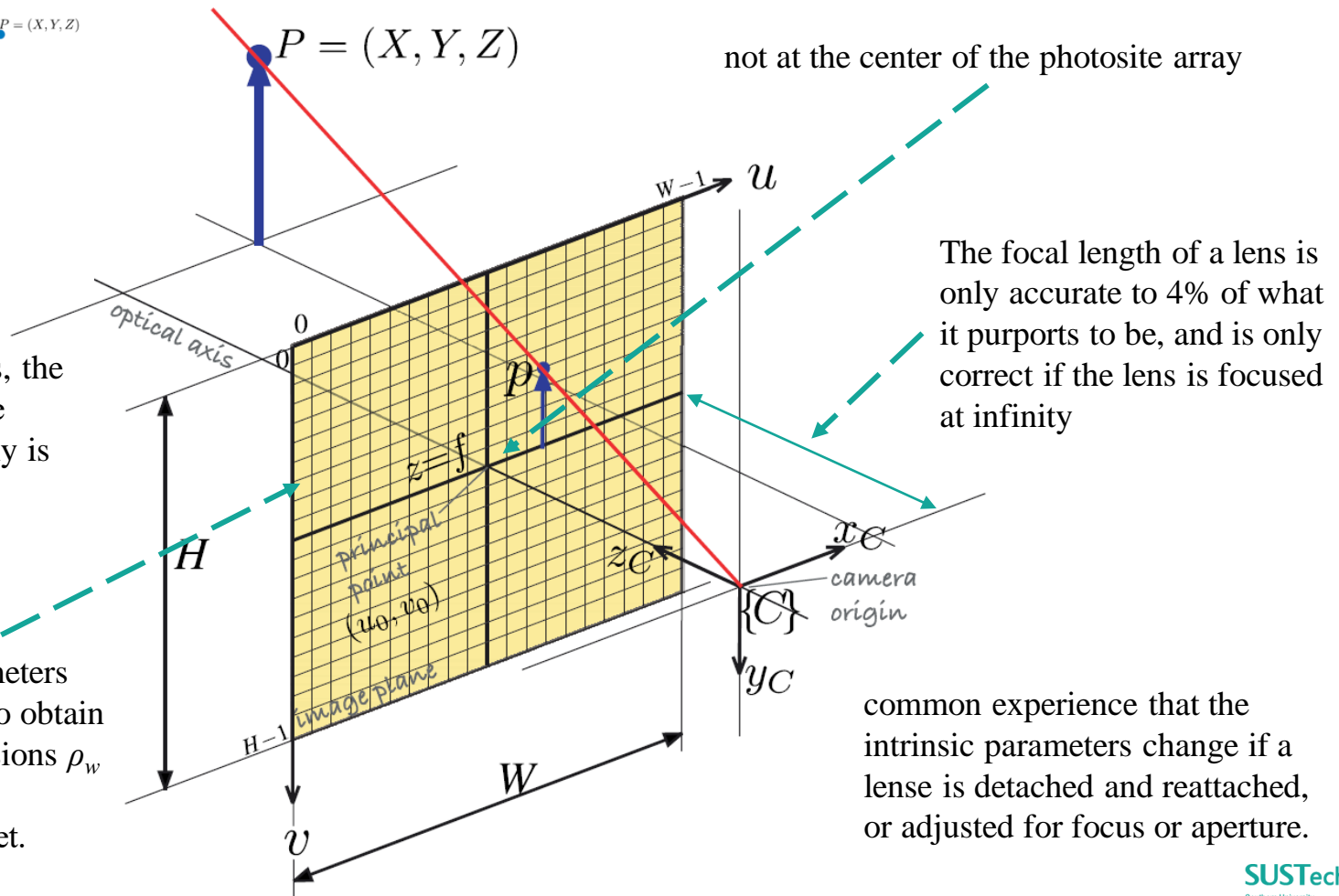
In general, the cameras are not made as modeled



The extrinsic parameters, the camera's pose, raises the question of where exactly is the center point of the camera.

The only intrinsic parameters that it may be possible to obtain are the photosite dimensions ρ_w and ρ_h from the sensor manufacturer's data sheet.

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Camera Calibration

Some are done before shipping, some are not, and some are provided with a software to do so

- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

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$$\tilde{p} = C\tilde{P}$$

Camera Calibration

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- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}}$$

$$\tilde{\mathbf{p}} = (u, v, 1)$$

$$u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

Camera Calibration

Some are done before shipping, some are not, and some are provided with a software to do so

- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}} \quad \begin{array}{l} \tilde{\mathbf{p}} = (u, v, 1) \\ u = \frac{u'}{w'}, v = \frac{v'}{w'} \end{array} \rightarrow \begin{array}{l} C_{11}X + C_{12}Y + C_{13}Z + C_{14} - C_{31}uX - C_{32}uY - C_{33}uZ - C_{34}u = 0 \\ C_{21}X + C_{22}Y + C_{23}Z + C_{24} - C_{31}vX - C_{32}vY - C_{33}vZ - C_{34}v = 0 \end{array}$$

Camera Calibration

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- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

Disregard overall scaling, set to 1

$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}} \rightarrow \begin{aligned} & \tilde{\mathbf{p}} = (u, v, 1) \\ & u = \frac{u'}{w'}, v = \frac{v'}{w'} \end{aligned} \begin{aligned} & C_{11}X + C_{12}Y + C_{13}Z + C_{14} - C_{31}uX - C_{32}uY - C_{33}uZ - C_{34}u = 0 \\ & C_{21}X + C_{22}Y + C_{23}Z + C_{24} - C_{31}vX - C_{32}vY - C_{33}vZ - C_{34}v = 0 \end{aligned}$$

Camera Calibration

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$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}} \quad \tilde{\mathbf{p}} = (u, v, 1)$$

$$u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

$$C_{11}X + C_{12}Y + C_{13}Z + C_{14} - C_{31}uX - C_{32}uY - C_{33}uZ - C_{34}u = 0$$

$$C_{21}X + C_{22}Y + C_{23}Z + C_{24} - C_{31}vX - C_{32}vY - C_{33}vZ - C_{34}v = 0$$

Increasing sampling for a solution

$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & \vdots & & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

Camera Calibration

Some are done before shipping, some are not, and some are provided with a software to do so

- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

Disregard overall scaling, set to 1

$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}} \quad \tilde{\mathbf{p}} = (u, v, 1)$$

$$u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

$$C_{11}X + C_{12}Y + C_{13}Z + C_{14} - C_{31}uX - C_{32}uY - C_{33}uZ - C_{34}u = 0$$

$$C_{21}X + C_{22}Y + C_{23}Z + C_{24} - C_{31}vX - C_{32}vY - C_{33}vZ - C_{34}v = 0$$

Increasing sampling for a solution

11 unknowns to be solved

$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & & \vdots & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

Camera Calibration

Some are done before shipping, some are not, and some are provided with a software to do so

- The process of determining the camera's intrinsic parameters and the extrinsic parameters with respect to the world coordinate system

Disregard overall scaling, set to 1

$$\tilde{\mathbf{p}} = \mathbf{C}\tilde{\mathbf{P}} \quad \tilde{\mathbf{p}} = (u, v, 1)$$

$$u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

$$C_{11}X + C_{12}Y + C_{13}Z + C_{14} - C_{31}uX - C_{32}uY - C_{33}uZ - C_{34}u = 0$$

$$C_{21}X + C_{22}Y + C_{23}Z + C_{24} - C_{31}vX - C_{32}vY - C_{33}vZ - C_{34}v = 0$$

Increasing sampling for a solution

11 unknowns to be solved

$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & \vdots & & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

$N \geq 6$ for a solution, but usually more are used to solve using *least square*

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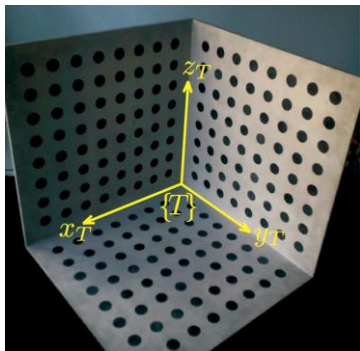
$$(u_i, v_i)$$

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$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & \vdots & & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

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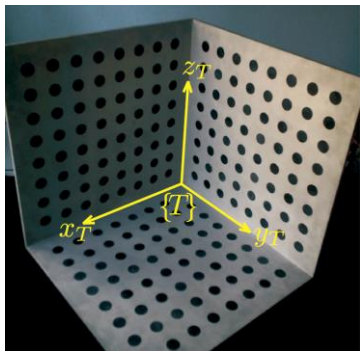
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$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & \vdots & & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

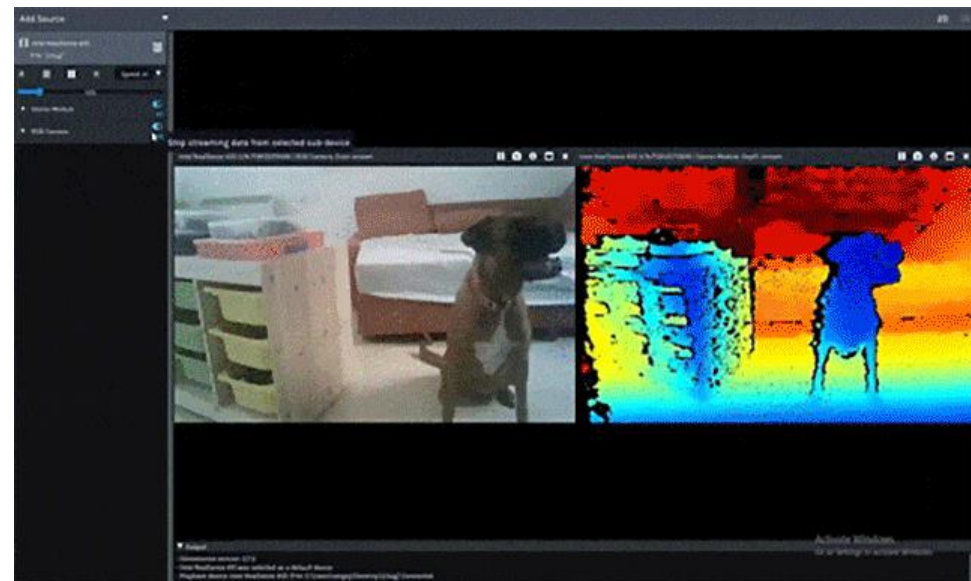
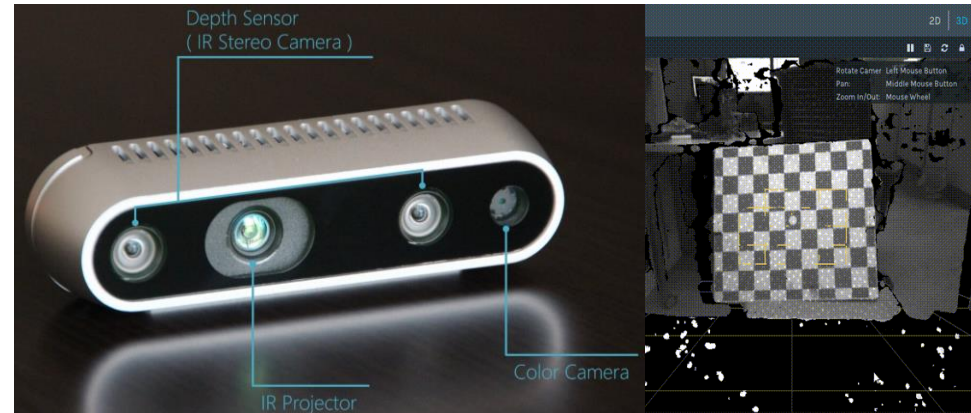
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What if the points are coplanar?

About Intel Realsense D435

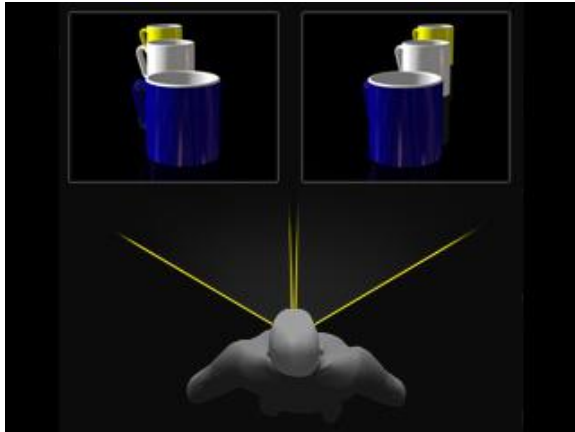
Entry level stereo depth sensor with abundant resources at a low cost

	Intel RealSense Depth Camera D435
Environment	Indoor and outdoor
Depth Technology	Active IR stereo
Image Sensor Technology	Global shutter: 3 um x 3 um pixel size
Main Intel® RealSense™ Products	Intel® RealSense™ vision processor D4 Intel® RealSense™ module D430
Depth Field of View (FOV)—(Horizontal × Vertical) for HD 16:9	85.2° x 58° (+/- 3°)
Depth Stream Output Resolution	Up to 1280 x 720
Depth Stream Output Frame Rate	Up to 90 fps
Minimum Depth Distance (Min-Z)	0.11 m
Maximum Range	Approximately 10 meters Accuracy varies depending on calibration, scene, and lighting conditions
RGB Sensor Resolution & Frame Rate	1920 x 1080 at 30 fps
RGB Sensor FOV (Horizontal × Vertical)	69.4° x 42.5° (+/- 3°)
Camera Dimension (Length x Depth x Height)	90 mm x 25 mm x 25 mm
Connector	USB Type-C*
Mounting Mechanism	One 1/4-20 UNC thread mounting point Two M3 thread mounting points



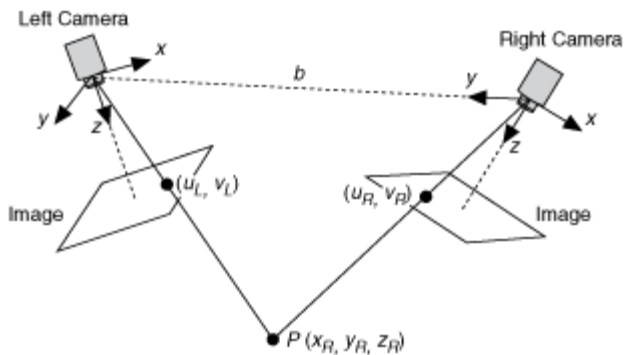
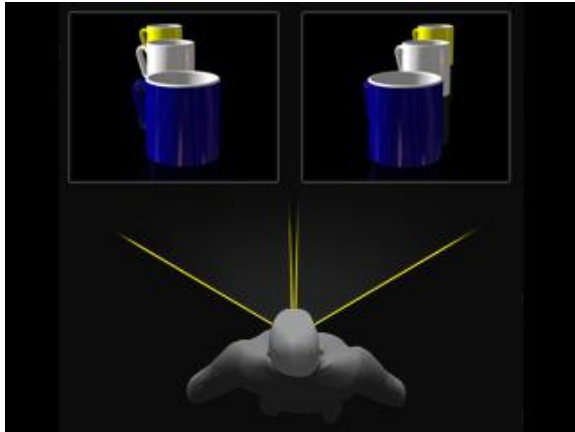
Stereo Vision

Triangulation Principle



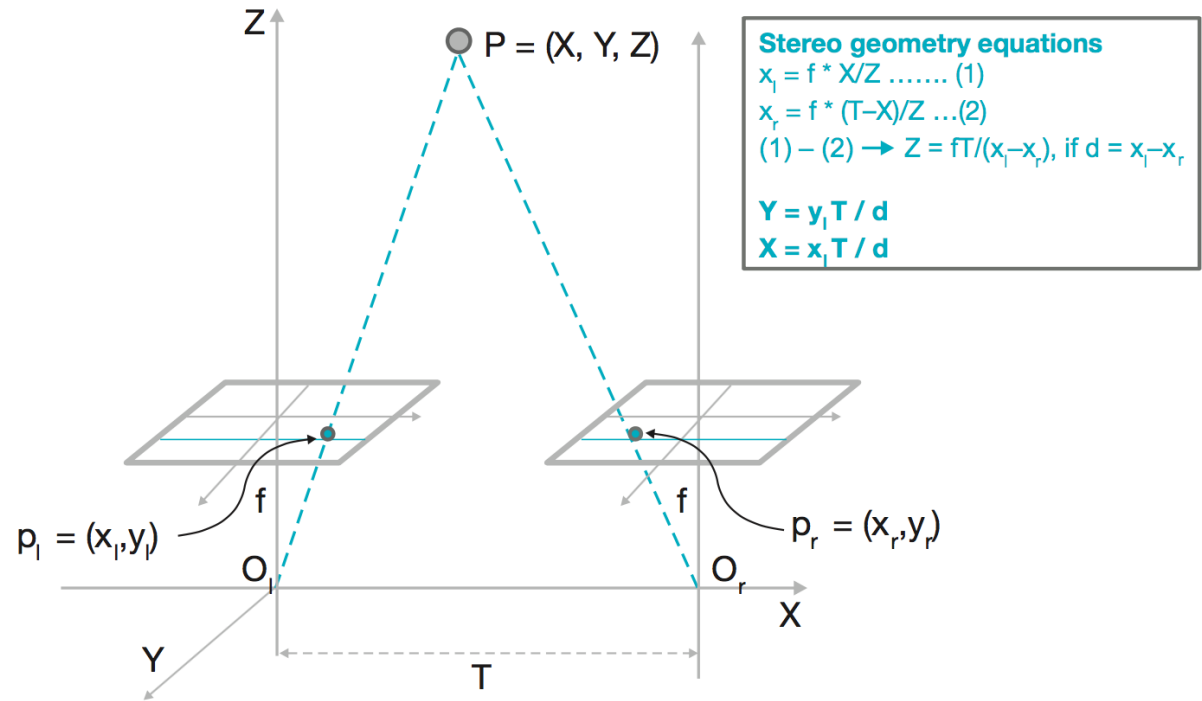
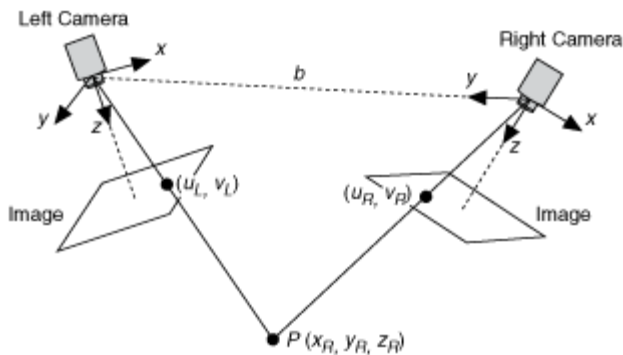
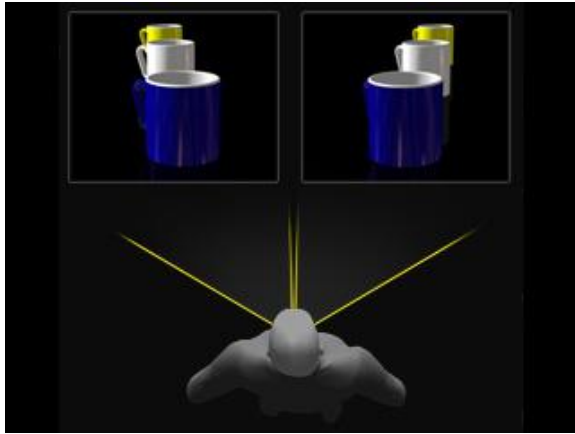
Stereo Vision

Triangulation Principle



Stereo Vision

Triangulation Principle

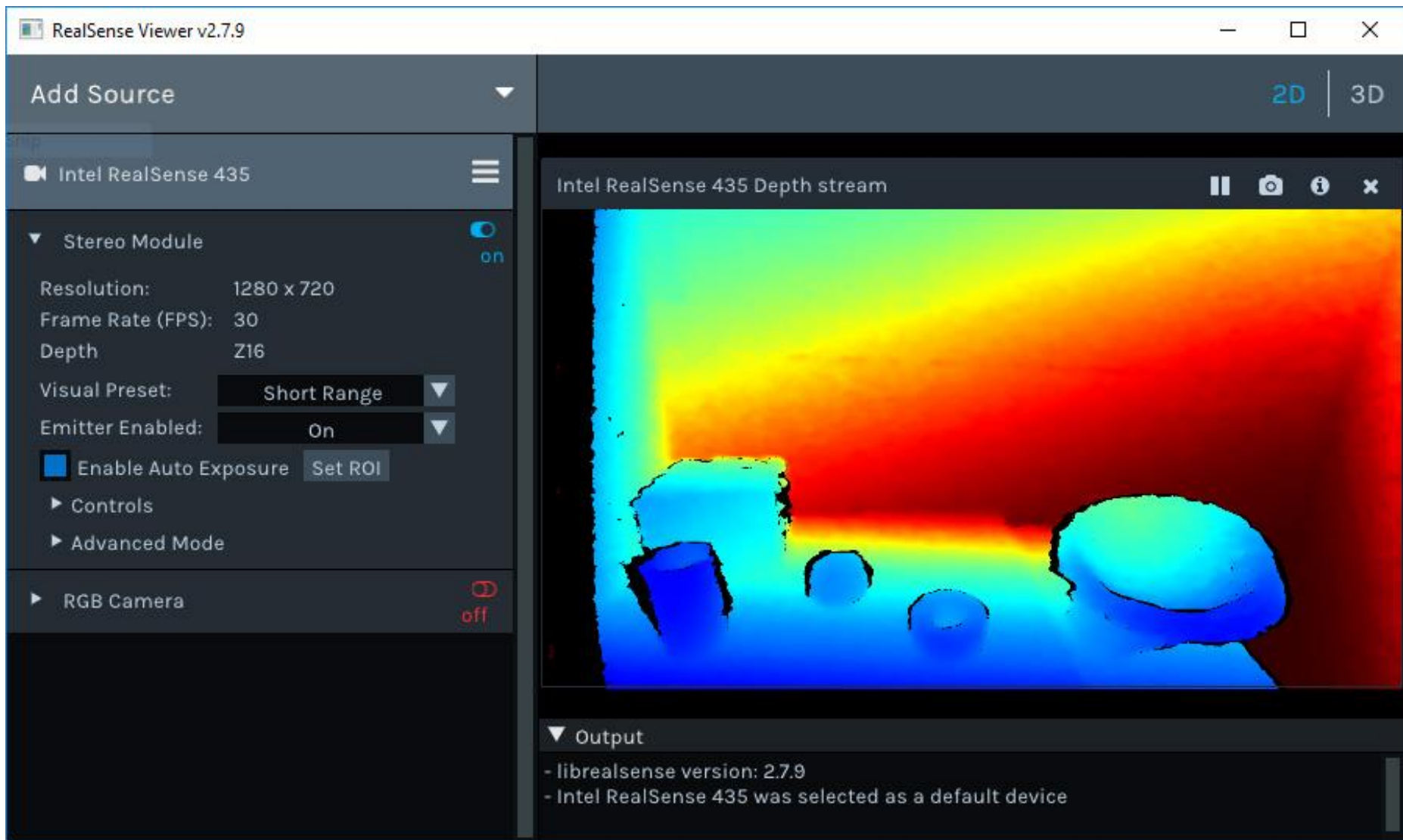


Interfacing Realsense with ROS

Installation

1. Connect the USB Type-C to your host PC.
2. Install the Intel® RealSense™ SDK V2.16(Please use the guidance here: <https://realsense.intel.com/sdk-2/#install>)
 1. Prep steps as instructed in the website
 2. `sudo apt-get install librealsense2=2.16.0-0~realsense0.85`
 3. `sudo apt-get install librealsense2-utils=2.16.0-0~realsense0.85`
 4. `sudo apt-get install librealsense2-dev=2.16.0-0~realsense0.85`
 5. `sudo apt-get install librealsense2-dbg=2.16.0-0~realsense0.85`
3. Run the Intel® RealSense™ Viewer (firmware upgrade may be needed if you encounter version error)

Interfacing Realsense with ROS



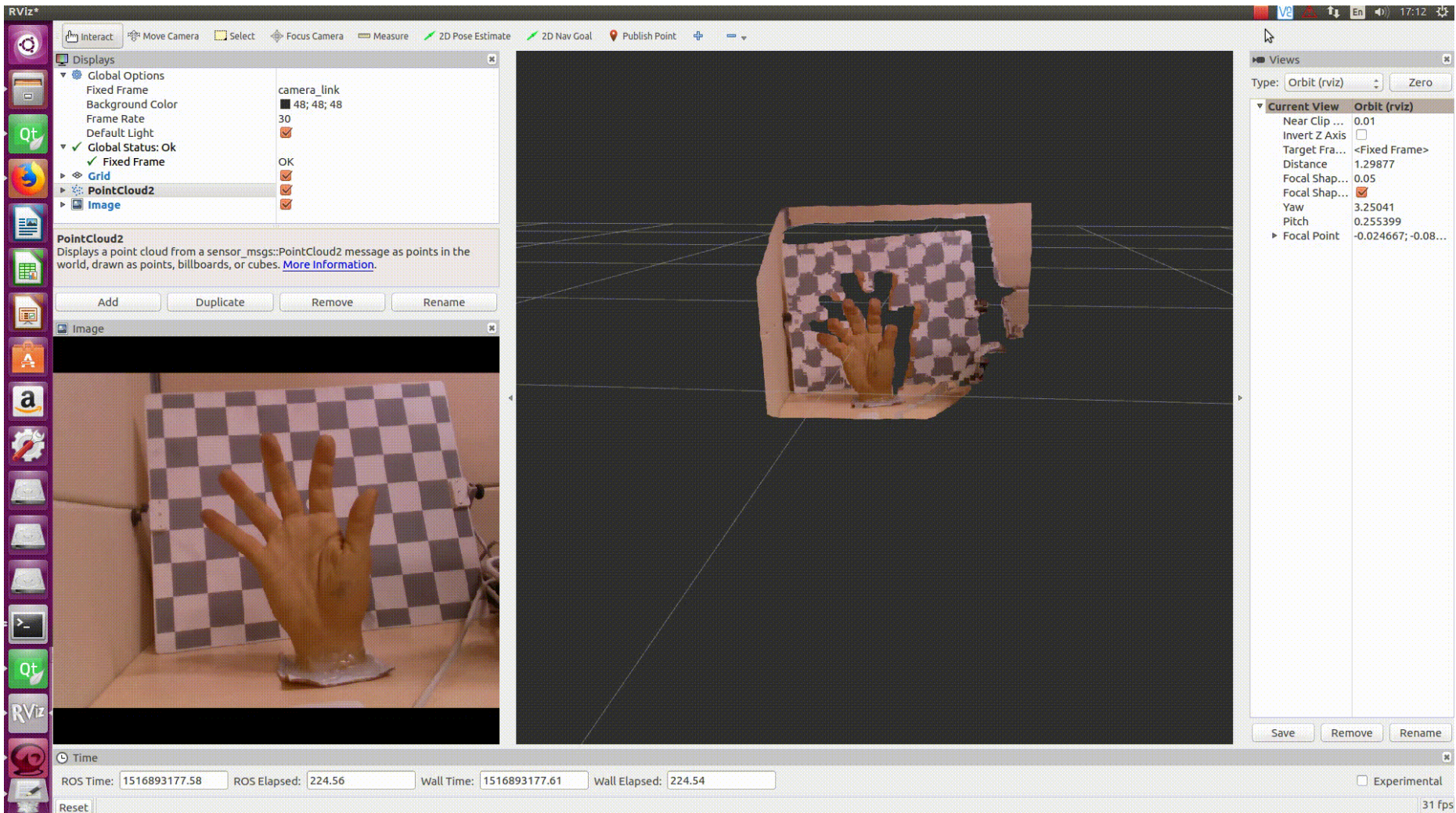
Interfacing Realsense with ROS

Realsense ROS Package

- Installation
 1. `cd ~/catkin_ws/src/`
 2. `git clone https://github.com/intel-ros/realsense.git`
 3. `git checkout tag/2.1.0`
 4. `catkin build & source ~/.bashrc`
- Launch: `roslaunch realsense2_camera rs_rgbd.launch`

Interfacing Realsense with ROS

Point cloud in Rviz



Interfacing Realsense with ROS

ROS topics published

```
bionicle@bionicle-mi:~/catkin_ws$ rostopic list
/camera/aligned_depth_to_color/camera_info
/camera/aligned_depth_to_color/image_raw
/camera/aligned_depth_to_color/image_raw/compressed
/camera/aligned_depth_to_color/image_raw/compressed/parameter_descriptions
/camera/aligned_depth_to_color/image_raw/compressed/parameter_updates
/camera/aligned_depth_to_color/image_raw/compressedDepth
/camera/aligned_depth_to_color/image_raw/compressedDepth/parameter_descriptions
/camera/aligned_depth_to_color/image_raw/compressedDepth/parameter_updates
/camera/aligned_depth_to_color/image_raw/theora
/camera/aligned_depth_to_color/image_raw/theora/parameter_descriptions
/camera/aligned_depth_to_color/image_raw/theora/parameter_updates
/camera/aligned_depth_to_infra1/camera_info
/camera/aligned_depth_to_infra1/image_raw
/camera/aligned_depth_to_infra1/image_raw/compressed
/camera/aligned_depth_to_infra1/image_raw/compressed/parameter_descriptions
/camera/aligned_depth_to_infra1/image_raw/compressed/parameter_updates
/camera/aligned_depth_to_infra1/image_raw/compressedDepth
/camera/aligned_depth_to_infra1/image_raw/compressedDepth/parameter_descriptions
/camera/aligned_depth_to_infra1/image_raw/compressedDepth/parameter_updates
/camera/aligned_depth_to_infra1/image_raw/theora
/camera/aligned_depth_to_infra1/image_raw/theora/parameter_descriptions
/camera/aligned_depth_to_infra1/image_raw/theora/parameter_updates
/camera/aligned_depth_to_infra2/camera_info
/camera/aligned_depth_to_infra2/image_raw
/camera/aligned_depth_to_infra2/image_raw/compressed
/camera/aligned_depth_to_infra2/image_raw/compressed/parameter_descriptions
/camera/aligned_depth_to_infra2/image_raw/compressed/parameter_updates
/camera/aligned_depth_to_infra2/image_raw/compressedDepth
/camera/aligned_depth_to_infra2/image_raw/compressedDepth/parameter_descriptions
/camera/aligned_depth_to_infra2/image_raw/compressedDepth/parameter_updates
/camera/aligned_depth_to_infra2/image_raw/theora
/camera/aligned_depth_to_infra2/image_raw/theora/parameter_descriptions
/camera/aligned_depth_to_infra2/image_raw/theora/parameter_updates
/camera/color/camera_info
/camera/color/image_raw
/camera/color/image_raw/compressed
```


Interfacing Realsense with ROS

ROS topics published

- `$rostopic ehco /camera/color/camera_info`

```
header:
  seq: 64
  stamp:
    secs: 1548158943
    nsecs: 184392163
  frame_id: "camera_color_optical_frame"
height: 480
width: 640
distortion_model: "plumb_bob"
D: [0.0, 0.0, 0.0, 0.0, 0.0]
K: [616.3788452148438, 0.0, 330.0303955078125, 0.0, 616.4257202148438, 234.33065795898438, 0.0, 0.0, 1.0]
R: [1.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 1.0]
P: [616.3788452148438, 0.0, 330.0303955078125, 0.0, 0.0, 616.4257202148438, 234.33065795898438, 0.0, 0.0, 0.0, 1.0, 0.0]
binning_x: 0
binning_y: 0
roi:
  x_offset: 0
  y_offset: 0
  height: 0
  width: 0
  do_rectify: False
---
```

Interfacing Realsense with ROS

subscribe to ROS topic of relevant image

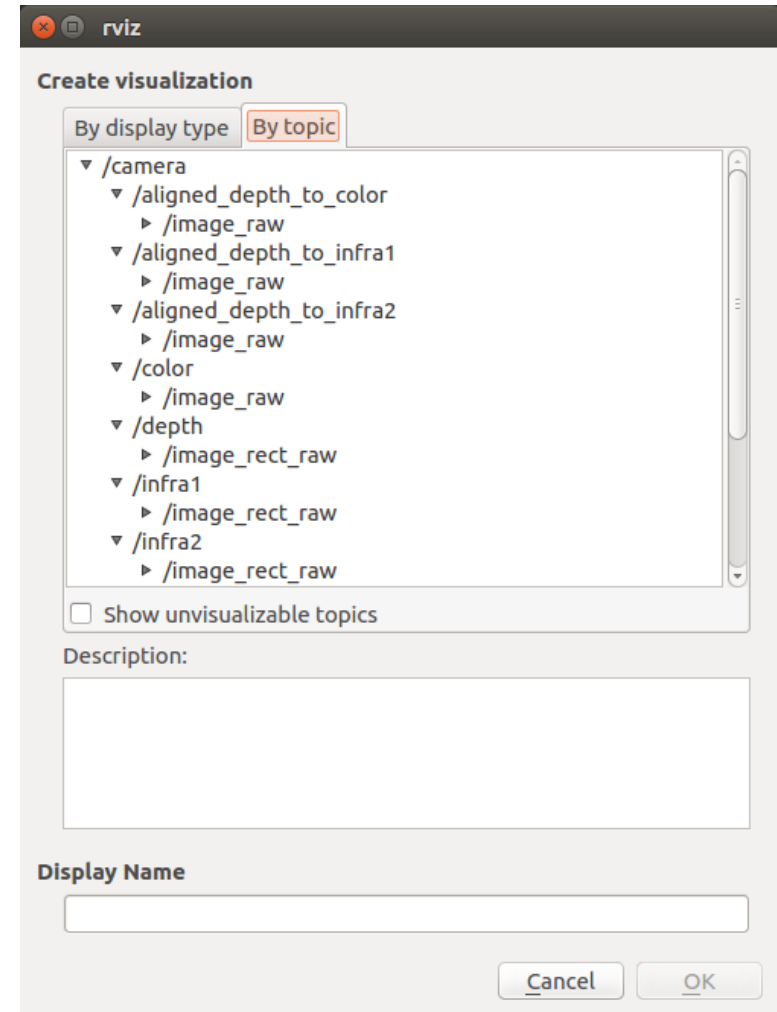
TODO: Create Subscribers

```
depth_sub =  
message_filters.Subscriber("/camera/aligned_depth_t  
o_color/image_raw", Image)
```

```
rgb_sub =  
message_filters.Subscriber("/camera/color/image_ra  
w", Image)
```

```
ts = message_filters.TimeSynchronizer([depth_sub,  
rgb_sub], 1)
```

```
ts.registerCallback(callback)
```



Homework

- Write a perception ros node subscribing to ros topic of relevant image.
- Add a mover function to move the robot arm in the perception ros node.
- Prepare for the next lab session:
 - Install opencv and pcl

Thank you!

Prof. Song Chaoyang

- Dr. Wan Fang (sophie.fwan@hotmail.com)

